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ABSTRACT

The learning of mathematics was addressed in this study, with attention in this first volume focused on defining specific learning styles in mathematics, classifying individuals according to learning styles, relating achievement to learning styles, associating learning styles with localized hemispheric functioning of the brain, correlating learning styles in mathematics with general learning styles, and tailoring teaching strategies to learning styles. The Mathematics Diagnostic/Prescriptive Inventory (MDPI), an individualized instrument assessing mathematical achievement, ability, and learning style, was administered to over 1,200 children, aged 5-18. Results of the testing led to the definition of two distinct mathematical learning styles. Comparing the findings of the MDPI with an exhaustive battery of neurological and neuropsychological tests administered to the same children established, clinically, associations between the child's mathematics learning style and hemispheric preference. A statistical study involving a wide variety of variables in mathematics, neurology, and neuropsychology coded for 220 subjects was also conducted. Results confirm that mathematics learning styles more strongly reflect qualitative aspects of performance than straight success or non-success. Conclusions from discriminant analyses conform closely to the assumption of two distinct learning styles. Examples of prescriptive teaching are included, as well as the MDPI and a sample case study. (MNS)

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MATHEMATICS LEARNING VIEWED FROM A NEUROBIOLOGICAL MODEL FOR INTELLECTUAL FUNCTIONING

FINAL REPORT: VOLUME ONE

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— PRINCIPAL INVESTIGATOR —

Grant Number: NIE-G-79-0089
Project Number: 9-0087

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December 1983

INTRODUCTION AND ACKNOWLEDGMENTS

This paper and the study it describes have been partially supported by grant G-79-0089 from the National Institute of Education, Teaching and Learning Program, with Patricia S. Davidson as Principal Investigator.

Other personnel associated with the project have been as follows:

Maria R. Marolda, Project Coordinator, Rater, Liaison with Children's Hospital Medical Center: Assistance in all aspects of the project and in the preparation of the final report.

Richard Truesdell, Statistical and Computer Consultant: Assistance in the statistical planning, execution, and analyses; and in the preparation of Section IX of the final report.

Paul A. Klein, Research Design Consultant: Assistance in the statistical research design, coordination, overview, and data interpretation; and in the preparation of the Reliability Study, Section IX, A.1 of the final report.

Joan Crooker, Research Assistant: Assistance in obtaining data from the hospital records and in the formation of independent files on each of the 220 subjects.

Ruth Marsal, Technical Secretary: Typing, art work and manuscript preparation for all aspects of the project and final report.

Special thanks go to each of the team members and also to the staff members of the Learning Disabilities Clinic, Department of Neurology, Children's Hospital Medical Center, Boston, Massachusetts for their help and cooperation: in particular, Martha Bridge Denckla, M.D. (Neurologist), director of the clinic during the three-year period, 1977-1979, in which the subjects being studied were tested; H.Harris Funkenstein, M.D. (Neurologist), current director of the clinic; and Jane Holmes, Ph.D., Neuropsychologist of the clinic.

Vital to this entire study are the children who have been evaluated over the past seven years with evolving versions of the mathematics diagnostic activities presented here. These close interactions with over 1,200 children, ages 5-18, together with the collaboration with the entire medical/educational

team also involved in the evaluation and discussion of the same children from their own specialized viewpoints (neurology, neuropsychology, psychiatry, audiology, speech and language, and reading) have been invaluable.

This final report is organized into eleven sections with Sections I - VIII bound in Volume I and Sections IX - XI bound in Volume II.

VOLUME I

Section I:	Purpose of Study: Description and Rationale
Section II:	Methods Used in the Study
Section III:	Results and Findings
Section IV:	Implications
Section V:	Prescriptive Teaching
Section VI:	Dissemination
Section VII:	Pedagogical Suggestions for Remediation and Enrichment
Section VIII:	Bibliography of Research on the Brain and Intellectual Functioning

VOLUME II

Section IX:	Statistical Analyses of Research Study
Section X:	Coding Key
Section XI:	Rating Manual for Coding Key

The author of this paper assumed responsibility for the overall directions of the project. All statements and interpretations presented herein are the author's and do not mean to suppose or suggest policies or beliefs of anyone else associated with the project, the University of Massachusetts, the National Institute of Education, or the Children's Hospital Medical Center.

MATHEMATICS LEARNING

VIEWED FROM A NEUROBIOLOGICAL MODEL

FOR INTELLECTUAL FUNCTIONING

by

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ABSTRACT

During the past seven years, clinical and statistical observations have pursued answers to the following questions:

(1) Can individuals be classifed according to specific learning styles in mathematics?

(2) Can specific learning styles in mathematics be defined by such factors as facility with particular mathematics topics or skills, ease with particular kinds of mathematical stimuli, style of processing, and effectiveness within different mathematical contexts?

(3) Are specific learning styles in mathematics associated with localized hemispheric functioning of the brain?

(4) Can teaching strategies be developed to respond to different learning styles?

The Mathematics Diagnostic/Prescriptive Inventory (MDPI), an individualized mathematics evaluation instrument assessing achievement, ability and learning style in mathematics, has been administered to over 1,200 children, ages 5 to 18, at the Learning Disabilities Clinic at the Children's Hospital Medical Center, Boston. The series of activities in the MDPI call upon those intuitions, skills, and strategies the child possesses for solving math problems. The child's approaches are as important as the answers.

Results of the testing led to the definition of two separate and distinct learning styles in mathematics. Comparing the findings of the MDPI with an exhaustive battery of neurological and neuropsychological tests administered at the clinic to the same children established clinically associations between the child's mathematics learning style and the child's hemispheric preference.

A thorough statistical study, involving a wide variety of variables in mathematics, neurology, and neuropsychology coded for the 220 subjects diagnosed at the clinic during a three-year period, was conducted under a research grant from the National Institute of Education. Results of the study confirm that learning styles in mathematics more strongly reflect qualitative aspects of performance than straight success or non-success. Conclusions from the discriminant analyses using the MDPI subtopics for predictor variables conform closely to the assumption of two distinct learning styles. It is of particular interest to find that failure to perform is often as valuable as success, and that "non-learned" characteristics derived from non-formal math activities or from behavioral postures are the most salient factors in reflecting mathematical and cognitive profiles.

An exciting result of this research is the direct applicability it offers for the development of teaching strategies compatible with distinct learning styles in mathematics. It is hoped that differentiated approaches to specific mathematical topics will make mathematics more accessible to students in the future.

TABLE OF CONTENTS FOR VOLUME I

PREFACE x - xiii

SECTION I: PURPOSE OF STUDY: DESCRIPTION AND RATIONALE

- Lines of Inquiry 1
- Neurobiological Model 2
- The Nature of Mathematics Learning 3
- The Mathematics Diagnostic/Prescriptive Inventory (MDPI):
  - General Overview 4
  - The Content 4
  - Behaviors Observed 7
  - The Evaluation Process 8
- Raw Data Recording Form 10
- Learning Styles in Mathematics:
  - Learning Style I
    - Achievement 33
    - "Math Sense" -- Problem Solving 34
    - Behaviors 34
    - Deficits 34
  - Learning Style II
    - Achievement 35
    - "Math Sense" -- Problem Solving 35
    - Behaviors 36
    - Deficits 36
- Mathematical Learning as it Relates to General Cognition 36
- Teaching Strategies Related to Learning Styles in Mathematics 37
- Overview of Mathematics Diagnostic Process 38
- The Integration of Diagnostic and Prescriptive Considerations through Case Studies: 45
  - Jennifer (Developmental Concern) 48
  - Aaron (Organizational Deficits) 51
  - Jay (Organizational Deficits, Imprecision, Problems with Preferred Learning Style) 54
  - Irwin (Integration Deficits and Emotional Concerns) 59

Table of Contents for Volume I, Cont.

Ellen (Generally Slow Progress) 64
Adam (Language Impairment) 67
Alex (Production Deficits, Organizational Deficits) 71
Julia (Math Learning Style I) 77
Michael (Organizational Deficits) 82
John (Math Learning Style II) 87
Paul (Lack of Confidence, Math Learning Style I) 91

SECTION II: METHODS USED IN THE STUDY

- Subjects 96
  - Sex Characteristics 97
  - Age Characteristics 98
  - Grade in School 99
  - Arithmetic Achievement as Reported by Teacher 100
  - Reported Previous Math Testing 101
  - Reported Results on Previous Math Testing 102
  - Reported Ability (WISC-R Previously Administered) 103
- Measures and Procedures 104
- Variables 114
- Overview of Research Design 122

SECTION III: RESULTS AND FINDINGS

- Demographic Findings 126
- Teacher-Reported Delays in Various Areas 127
- Clinically Generated Demographical Findings 128
- Neuropsychological Findings Clearly Reported 129
- Learning Styles in Mathematics:
  - Preferred Learning Style 130
  - Impaired Learning Style 131
- Findings Generated from Rating Techniques for Neurology, Neuropsychology, and Mathematical Learning Styles 132

Table of Contents for Volume I, Cont.

SECTION IV: FINDINGS & IMPLICATIONS OF THE STATISTICAL STUDY

- Findings Related to Factor Analyses 134

  Factor Analysis on MDPI Subtopics 134
  Factor Analysis on Subjects Greater Than 9 Years of Age 136
  Factor Analysis on Selected MDPI Items Considered to be Contributory to Learning Style in Mathematics 137

- Implications from Factor Analyses 138
- Findings Related to Discriminant Analyses 138

  Levels of Prediction from Discriminant Analyses 140

- Implications from Discriminant Analyses 142
- Individual Components that Appeared as Contributors to Four or More Predictive Constellations 148
- General Implications from the Totality of Discriminant Analyses 149
- Heuristic Issues 150

SECTION V: PRESCRIPTIVE TEACHING

- Formulating Effective Teaching Strategies 152
- Examples of Prescriptive Teaching Using a Neurobiological Framework for Mathematics Learning 154

  Upper Decade Addition Facts 154
  Upper Decade Subtraction Facts 158
  Place Value 161
  Multiplication Facts 167
  Multiplication of One-Digit Times Two-Digit Numbers 180
  Multiplication of Two-Digit by Two-Digit Numbers 185

SECTION VI: DISSEMINATION

- Résumé of Patricia S. Davidson, Principal Investigator 191
- Dissemination -- Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education: 1979-1983 202

SECTION VII: PEDAGOGICAL SUGGESTIONS FOR REMEDIATION AND ENRICHMENT

- Introduction 218
- Counting and Number Work 220
- Arithmetic Operations with Special Emphasis on Place Value and Concept Development 222
- Games and Puzzles to Practice Arithmetic Concepts and Operations 224
- Geometric Concepts 228
- Fraction Concepts 234
- Decimal Concepts 236
- Logical Thinking and Problem Solving 237
- Further Enrichment Activities 241
- Problem Solving and Applications with Calculators 243
- Algebraic Concepts 244

SECTION VIII: SELECTED BIBLIOGRAPHY OF RESEARCH ON THE BRAIN AND INTELLECTUAL FUNCTIONING

- Introduction 246
- Selected Bibliography of Research on the Brain and Intellectual Functioning 247

SECTIONS IX, X, XI: [See Table of Contents for Volume II, following]

TABLE OF CONTENTS FOR VOLUME II

SECTION IX: STATISTICAL ANALYSIS OF RESEARCH STUDY

A.0 Statistical Analysis 1

A.1 Reliability: Analysis of Research Design -- Section 1 5

Reliability 4
Reliability of Ratings 5

A.2 Exploratory Factor Analysis: Analysis of Research Design -- Section 2 7

A.2.1 Factor Analysis of MDPI Variables -- Overall 7

Table A.2.1.1: Factor 1 8
" " Factor 2 9
" " Factor 3 10
" " Factor 4 11
" " Factor 5 11
Table A.2.1.2 12
Table A.2.1.3 13

A.2.2 Factor Analysis of MDPI Variables -- Age Groups 14

Table A.2.2.1: Factor 1 15
" " Factor 2 15
" " Factor 3 15
" " Factor 4 16
" " Factor 5 16
Table A.2.2.2 16
Table A.2.2.3 17

A.2.3 Factor Analysis of MDPI Variables -- Within Clinically Identified Learning Style Groups 18

A.2.4 Factor Analysis of MDPI Variables -- Other Subsets of Interest 18

Table A.2.4.1: Factor 1 18
" " Factor 2 19
" " Factor 3 19
" " Factor 4 19
" " Factor 5 20

Table of Contents for Volume II, Cont.

Table A.2.4.2 20
Table A.2.4.3 21

A.3 Exploratory Discriminant Analysis: Analysis of Research Design -- Section 3 22

Discriminating Variables 24

A.3.1 Discriminant Analysis of Area 1 25

A.3.1.1 - Discriminant Analysis of Left Hemispheric Function (V44) Using MDPI Subtopics as Discriminating Variables 26

A.3.1.2 - Discriminant Analyses of Left Frontal Function (V46) Using MDPI Subtopics as Discriminating Variables 27

A.3.1.3 - Discriminant Analysis of Right Frontal Function (V47) Using MDPI Subtopics as Discriminating Variables 28

A.3.1.4 - Discriminant Analysis of Left Hemispheric Function (V70) Using MDPI Subtopics as Discriminating Variables 28

A.3.1.5 - Discriminant Analysis of Right Hemispheric Function (V71) Using MDPI Subtopics as Discriminating Variables 31

A.3.2 Discriminant Analysis of Area 2 34

A.3.2.1 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style I Preferred (V208) Using MDPI Topics as Discriminating Variables 34

A.3.2.2 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style II Preferred V209) Using MDPI Subtopics as Discriminating Variables 37

A.3.2.3 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style I Impaired (V210) Using MDPI Subtopics as Discriminating Variables 40

Table of Contents for Volume II, Cont.

A.3.2.4 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style II Impaired (V211) Using MDPI Subtopics as Discriminating Variables 42

A.3.3 Discriminant Analysis of Area 3 45

A.3.3.1 - Discriminant Analysis of Left Hemispheric Function (V44) Using MDPI Rater Observations and Conclusions as Discriminating Variables 45

A.3.3.2 - Discriminant Analysis of Left Frontal Function (V46) Using MDPI Rater Observations and Conclusions as Discriminating Variables 48

A.3.3.3 - Discriminant Analysis of Right Frontal Function (V47) Using MDPI Rater Observations and Conclusions as Discriminating Variables 51

A.3.3.4 - Discriminant Analysis of Left Hemispheric Function (V70) Using MDPI Rater Observations and Conclusions as Discriminating Variables 54

A.3.3.5 - Discriminant Analysis of Right Hemispheric Function (V71) Using MDPI Rater Observations and Conclusions as Discriminating Variables 57

A.3.4 Discriminant Analysis of Area 4 60

A.3.4.1 - Discriminant Analysis of Left Hemispheric Function (V44) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 60

A.3.4.2 - Discriminant Analysis of Left Frontal Function (V46) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 63

A.3.4.3 - Discriminant Analysis of Right Frontal Function (V47) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 66

Table of Contents for Volume II, Cont.

A.3.4.4 - Discriminant Analysis of Left Hemispheric Function (V70) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 69

A.3.4.5 - Discriminant Analysis of Right Hemispheric Function (V71) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 72

A.3.5 Discriminant Analysis of Area 5 75

A.3.5.1 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style I Preferred (V208) Using MDPI Rater Observations as Discriminating Variables 75

A.3.5.2 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style II Preferred (V209) Using MDPI Rater Observations as Discriminating Variables 78

A.3.6 Discriminant Analysis of Area 6 81

A.3.6.1 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style I Preferred (V208) Using MDPI Subtopics and Rater Observations as Discriminating Variables 82

A.3.6.2 - Discriminant Analysis of the MDPI Rater Conclusion of Mathematics Learning Style II Preferred (V209) Using MDPI Subtopics and Rater Observations and Conclusions as Discriminating Variables 84

A.3.7 Summary of the Results of Section 3 87

Table Summarizing Results of Section 3 88

A.3.7.1 - Table Listing all MDPI Variables and their Involvement in Discriminant Functions for Associated Group Variables 91

Table of Contents for Volume II, Cont.

SECTION X: CODING KEY

- General Data (Card 01) 111
- Neuropsychological Data (Card 02) 118
- Neurological Data (Card 03) 123
- Math Profile Data (Card 04) 128
- Math Profile Data (Card 05) 135
- Math Profile Data (Card 06) 141

SECTION XI: RATING MANUAL FOR CODING KEY

- General Data (Card 01) 149
- Neuropsychological Data (Card 02) 164
- Neurological Data (Card 03) 188
- Mathematical Data (Cards 04, 05) 201
- Mathematics Findings (Card 06) 230

SECTION I

PURPOSE OF STUDY: DESCRIPTION AND RATIONALE

- LINES OF INQUIRY
- NEUROBIOLOGICAL MODEL
- THE NATURE OF MATHEMATICS LEARNING
- THE MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY
- RAW DATA RECORDING FORM
- LEARNING STYLES IN MATHEMATICS
- MATHEMATICS LEARNING AS IT RELATES TO GENERAL COGNITION
- TEACHING STRATEGIES RELATED TO LEARNING STYLES IN MATHEMATICS
- OVERVIEW OF MATHEMATICS DIAGNOSTIC PROCESS
- THE INTEGRATION OF DIAGNOSTIC AND PRESCRIPTIVE CONSIDERATIONS THROUGH CASE STUDIES

SECTION I

PURPOSE OF STUDY: DESCRIPTION AND RATIONALE

Lines of Inquiry

The learning of mathematics was addressed in this study with specific focus on the following issues:

- Can specific learning styles in mathematics be defined? Can these learning protocols be differentiated with respect to
  - facility in particular mathematical topics or skills,
  - ease with particular kinds of mathematical stimuli (concrete, pictorial, or abstract)
  - style of processing in mathematical activities (inductive, deductive)
  - applications within various contexts (arithmetic, algebraic, geometric, or logical).
- Can individuals be classifed according to specific learning styles? Can individuals who achieve (or do not achieve) at the same level of mathematical achievement arrive at those skills through different learning styles?
- Are the specific learning styles in mathematics associated with localized hemispheric functioning of the brain?
- Do particular learning styles in mathematics reflect general learning styles?
- Can teaching strategies be tailored in response to different learning styles?

Neurobiological Model

Although many frameworks for cognition have been offered, the neurobiological model was chosen since it emphasizes the differential aspects of learning, just as the proposed hypotheses emphasize the differential aspects of mathematical learning styles.

The neurobiological model relates the various aspects of learning to the specific aspects of the central nervous system, such as the cerebral hemispheres, and corpus callosum.

Although the neurobiological model still leaves much to be explored, there are some general principles that are well established. The left hemisphere of the brain is primarily concerned with processing discrete elements in a series (one at a time), is strongly oriented toward details, is analytical, and plays a major role in verbal communication. The right hemisphere is concerned more with the overall configuration (the gestalt), is spatially oriented, is intuitive, and is adept at visual communication (such as reading faces and body expressions). It is also known that, with respect to the motor system, the left hemisphere has primary responsibility for the right side of the body; while the right hemisphere has primary responsibility for the left side of the body.

The membrane between the two hemispheres, known as the "corpus callosum," carries on important and rapid communication between the two hemispheres. (Major impetus for recent study into the lateralization of brain functioning has come from the so-called "split brain" patients whose corpus callosums were entirely severed, leaving the two cerebral hemispheres totally disconnected.)

The neurologist looks at both the gross and fine motor outputs of the central nervous system, especially the particular manifestations of the spinal cord and how well it communicates (sends and receives) messages to and from the brain. Some exercises which the neurologist typically presents involve the observation of running, hopping, and balancing activities; testing of reflexes; checking on problems with speech muscles; assessing sensory dysfunction; checking on effectiveness of patterned movements; and observing various asymmetries in function on the left and right sides of the body.

Purpose of the Study: Description and Rationale, Cont.

The neuropsychologist looks at intellectual or cognitive outputs of the central nervous system. The neuropsychologist infers functioning of the central nervous system in general, and the brain in particular, through observation of "thinking behaviors." Typical exercises of the neuropsychologist include an individualized intelligence test, such as the WISC, with particular note taken of differences in attainment on verbal tasks (information, comprehension, similarities, vocabulary), and performance tasks (picture completion, block design, object assembly, and picture arrangement).

The neuropsychologist may also administer the Visual Motor Integration Test (VMI), and the Rey-Osterreich Complex Figure Design Copying Task, which measures among other things, organizational aspects, such as alignment of parts, intersection of lines, proportion, and stylistic aspects, such as presence or absence of external configuration, directionality, and continuity/discontinuity of lines. Other tests include the Ravens Progressive Matrices, which consists of highly perceptually-oriented activities, and tasks involving non-standard sequences, using the days of the week, months of the year, the alphabet, and digit patterns. (Examples of the latter would be to say only the letters of the alphabet that rhyme with "tree" or to repeat a digit pattern backwards.)

Furthermore, the brain is also viewed by both the neurologist and neuropsychologist in its back-front dimension, which according to a grossly over-simplified interpretation defines "input-output" systems. The posterior (back) areas of the brain receive and integrate information; the massive frontal systems are the major organizers of behavior in that they select, control, inhibit, or organize output. The interface between these two functions and the specialization of the hemispheres make the neurobiological implications of learning extremely interesting and complex.

The Nature of Mathematics Learning

In order to embark on an analysis of how children learn mathematics, a very specific definition of those concepts and skills which are encompassed in the study of mathematics must be made. In this study, mathematics is viewed from many perspectives. Mathematics is seen as a variety of topics, some related (e.g. addition and multiplication) and some apparently not

related (e.g. counting and symmetry); as contained in a variety of contexts (arithmetical, geometric, algebraic, and logical); as manifest in a variety of modalities (concrete, pictorial, and abstract); and as process as well as specific solutions. Mathematics is generally approached then as a "way of thinking," a way a interacting with various aspects of our environment.

The Mathematics Diagnostic/Prescriptive Inventory was selected to assess all of these various aspects of mathematics, particularly the processes leading to responses. This instrument was specifically designed to explore the fullness of mathematics learning.

The Mathematics Diagnostic/Prescriptive Inventory (MDPI) General Overview

The Mathematics Diagnostic/Prescriptive Inventory (MDPI) is an individualized mathematics evaluation instrument designed to assess achievement, ability, and learning style in mathematics.

The purpose of the MDPI is to provide situations in which mathematical thinking can be assessed. The examiner presents carefully designed and sequenced activities in order to elicit the intuitions, skills, and strategies which the examinee possesses or has acquired. Various mathematical contexts consisting of arithmetic, algebraic, geometric, and logical situations are presented. The examiner compares and contrasts the responses in each context.

The Content

The activities presented include most skills and procedures typical to traditional mathematics curricula. However, other important skills and procedures are included. The activities are offered in three modalities: concrete, pictorial, and abstract. The individual may approach each task in any of these three modalities. Furthermore, responses may be given in any of these same modalities. The examiner notes the medium which the individual spontaneously selects as well as the one in which he/she appears most comfortable. Thus, qualitative aspects of the solution process as well as the quantitative responses are assessed.

Oral counting activities are presented. Children are asked to count forwards, count on (count forward from a number other than 1), and count backwards. Older children are asked to count by 2's, 5's, and 10's. Since oral counting underlies all arithmetic processes, these skills can be used as primitive approaches to all arithmetic.

Activities involving symbolic representation of numerals and shapes are presented. The child is asked to read and write numerals. Reversals of single symbols as well as digits is assessed. Numerical problems are dictated. The child's recall of the elements of the problem, use of operational symbols (+, -, x, ÷) and organization of the elements of the problem are evaluated.

Basic, quantitative concepts are assessed. Activities involving the manipulation of plastic chips evaluate the development of the concepts of cardinality of sets (associating a quantity with a group of objects), one-to-one correspondence, conservation of number, and ordinality of number (first, second ...), as well as concepts of "more, less, one more, one less, between." The concepts of intuitive place value are also evaluated.

Many (45) standard arithmetic problems are presented. The responses are noted as well as the ability to recall facts and to recall multi-step procedures (without the need for facts). The level of problem and operation satisfactorily completed are noted as well as the ability to interpret the operation in a real context (i.e. understanding of the basic concept itself). The need for concrete materials is assessed. The organization and work style during the solution procedure are observed.

Some simple geometric concepts are assessed. Familiarity with basic shapes is assessed. Activities involving figure/ground discrimination in geometric designs as well as visual memory are presented.

Concepts and operations involving estimation, serial mental computation, fractions, decimals, integers, and algebraic skills are presented to older children.

Problem solving is presented in a variety of situations. The goal in these activities is to assess an individual's "math sense" as seen in the ability to distill key information, ability to formulate approaches to problems,

ability to apply and pursue reasonable strategies, and the ability to apply logical thinking. The need for computational skills to find solutions is minimized.

Flexibility of thought and ability to learn new concepts or procedures are also assessed. The situations presented are varied. Individuals are asked to do those sections which seem appropriate to their age, previous experiences, and time constraints. The specific categories of situations are described below.

Real-life applications are orally described. The situations call upon pre-addition, pre-multiplication, pre-division, pre-fractions, and ratio concepts. The minimal prerequisite is skill in counting. Concrete materials, pencils and paper or "mental manipulation" are allowed. The modality most comfortably utilized is noted.

Pictorial graphs, isographs, and line graphs are given. These graphing tasks assess the ability to decode symbols, read data, interpret data, and extrapolate new information. The cues are pictorial. The questions and responses are oral.

Problem solving activities related to the basic concept of number, including place value and arithmetic operations are presented. Situations are presented using both discrete and continuous models for number. A child's preference is noted.

Analysis of simple sequences of chips, wooden shape blocks, pictures, and numerals is required. These activities assess the ability to compare elements, to give the next elements, and to describe the governing pattern. For older children, pairs of numbers (intuitive functions) are analyzed for the next pair in the pattern and for the governing rule.

Analysis of pictures of sets of designs for key (defining) characteristics (Elementary Science Study Creature Cards) are also included. Sets with one defining attribute, two simultaneously defining attributes, or more than three defining attributes are presented.

Problem solving on a geoboard leads to informal procedures for finding areas of triangles and polygons. This activity is presented to older or more sophisticated children. It is noted if a child does more or less well in problem solving in the geometric context than in an arithmetic one or algebraic one.

Since the preceding activities are presented as games or puzzles, the child's reaction to seemingly non-mathematical activities is compared to his attitudes during the more recognizable mathematical tasks. Preferences for kinds of activities -- arithmetic, algebraic, geometric or logical -- as well as preferences for modalities -- concrete (discrete or continuous models), pictorial or abstract -- are noted. In unfamiliar activities, the student may actually be taught a new concept or procedure. The child's "ability to learn" is informally assessed.

Behaviors Observed

During these activities, an individual's particular learning style in mathematics is observed. Some salient aspects of an individual's posture in mathematics include

- Logical thinking by a step-by-step (deductive) procedure vs. a gestalt (inductive) approach
- Degree of use and need for concrete materials
- Preference for discrete (set) approaches vs. continuous (length) approaches
- Flexibility of thought
- Ability to learn new materials, concepts, or procedures
- Behavioral contributions or interferences:

  need for physical movement

  perseveration

persistence

attention span

ability to focus

need for structured environment

- Self-image and confidence with regard to mathematical skills

Memory skills are also entwined with mathematical success. The specific kinds of recall skills within mathematical contexts include

- Memory for arithmetic facts
- Memory for multi-step procedures
- Memory for salient mathematical components of real-life situations
- Memory for directions
- Visual memory

The Evaluation Process

The items in the Mathematics Diagnostic/Prescriptive Inventory span the age range of 5-18 years. The role of the examiner is vital in eliciting and noting key skills and behavior of the individual in selecting appropriate activities from the suggested ones. The particular items used depend on the examinee's age, prior experiences, and responses to previous areas of the test.

The examiner evaluates responses on both objective and qualitative bases, noting both the actual solution and the process leading to it. The apparent degree of stability of the concepts or learned procedures is also assessed.

Strengths and deficits as demonstrated by responses to particular items are examined individually and as a group. The constellation of

successes or weaknesses yields suggestions about learning preferences and future expectations. The examiner reports the characteristics that may place an individual at risk for future success in a particular topic in mathematics or in mathematics in general and postulates the conditions and/or potential for growth in mathematics.

The Mathematics Diagnostic/Prescriptive Inventory yields diagnostic impressions, personalized recommendations, and specially tailored pedagogical suggestions for individuals. The prescriptions are based on the profile generated by the individual's performance on a wide variety of items selected from this activity-based mathematical inventory. The examiner assesses the individual's particular strengths, weaknesses, and learning style in order to plan carefully for future growth in mathematics.

The raw data for the Mathematics Diagnostic/Prescriptive Inventory is recorded on the following form. The examiner is encouraged to use the blank space to record observations and descriptions of the processes used.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY
RAW DATA RECORDING FORM

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I. ORAL COUNTING

A. Counting forward
B. Counting on from _____
C. Counting backwards from _____
D. Skip count by:

1. 10's:
2. 5's:
3. 2's:
4. 10's starting at 7:

Note: Use of Rhythm _____
Omission of Elements _____
Extensions beyond 100 _____

II. BASIC QUANTITATIVE CONCEPTS

A. Concept of Quantity

(Use Counting Chips)

1. "Give me 6 chips."

(Use Flip Book)

2. "Give me the same number of chips as dots on the paper."

a. X _____
b. Z _____
c. □ _____
d. Random _____
e. 5 x 5 _____

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II. Basic Quantitative Concepts, cont'd.

(Use Flip Book and Numeral Cards)

3. "Match each dot card with the appropriate numeral card."

a. X : 5 _____ d. Random: 12 _____
b. Z : 11 _____ e. 5 x 5: 25 _____
c. ☐ : 12 _____

B. Conservation of Number

(Use Counting Chips)

1. "Put out the same number of chips as I have." _____
2. Spread out the second set. "Which has more? Why?" _____
3. Stack up second set. "Which has more now? Why?" _____
4. Put one set under the table. "Which has more? Why?" _____
5. Give two sets of chips. (12, 14) "Which has more? _____
How do you know?" _____

Note Technique Used:

One-by-one Matching _____
Matching Lengths _____
Counting _____

C. One-to-One Correspondence

(Use Attribute Blocks)

1. "Sort the blocks into two groups: large ones and small ones." _____
2. "Can you tell me if there are more large ones or more small ones? _____
3. "Prove to me which group has more blocks, without counting." _____

Note Technique Used:

Gross One-by-One Matching (Any Large with Any Small) _____
Specific One-by-one Matching (Each Large with Its Small) _____
Physical Appearance _____
Towers _____
Comparison by Subsets of Shapes _____
Comparison by Subsets of Colors _____

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II. Basic Quantitative Concepts, cont'd.

D. Comparison of Numbers

1. "Tell me a number that is:

a. more than _____ . _____
b. less than _____ . _____
c. one more than _____ . _____
d. one less than _____ . _____
e. between 8 and 9. _____ .
f. between 8 and 8 1/2 _____ .
g. between 8 and previous answer. _____

E. Ordinality of Number

(Use Flip Book)

a. the first flower _____
b. the third flower _____
c. "What is this flower called?" (4th) _____

F. Intuitive Place Value

1. Using Cuisenaire Rods

a. "Put these 10 rods in order from smallest to largest." _____
b. "How many white rods match the black rod?" _____
c. "How many white rods match the blue rod?
White rods cannot be placed anywhere near the blue rod.
All other rods can be used and moved." _____

Note Technique Used:

Use of Finger(s) to "Measure" _____
Use of the Black Rod _____
Use of Other-Colored Rods _____
Counting the Rods in the Staircase _____
Attempt to Compare the Blue Rod with the White Rod _____
Another Method _____

2. Using Counting Chips

a. "Pretend these chips are money.
Pretend the blue chip is a dime.
Pretend the yellow chip is a penny.
How many pennies can be traded for a dime?" _____

b. "Show me 23¢. _____
"Another way?" _____

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II. Basic Quantitative Concepts, cont'd.

F. Intuitive Place Value, cont'd.

3. Using Base 10 Blocks

a. "Look at this wood.
There are three kinds of pieces." (Demonstrate)

b. "Show me a collection worth 26." _____
Another way?" _____

Note:

Competence with Discrete Models _____
Competence with Continuous Models _____
Preferred Model _____
Place Value Concepts Formulated _____
Flexibility of Thought _____

III. Numerical Symbolism

(Give individual paper and pencil.)

A. "Write:

6 17 23 47 462 3289"

B. "Read these numerals: "

(Use Flip Book)

5 13 37 362 101 5712 20315

C. "Write these problems. You do not need to find the answers."

1. "7 + 2 = "
2. "9 - 6 = "
3. "2 + 3 - 5"

"Use vertical notation to write these problems."

4.
$$\begin{array}{r} 17 \\ +\ 23 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 19 \\ +\ 3 \\ \hline \end{array}$$

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6. $\begin{array}{r} 12 \\ -\ 4 \\ \hline \end{array}$

7. $\begin{array}{r} 80 \\ 5 \\ +\ 16 \\ \hline \end{array}$

Note:

Request for Repetitions _____
Use of Reauditorization _____
Recall Skills _____
Use of Operation Signs _____
Organizational Skills _____
Reversals _____
Lining up of Place Values _____
Omissions _____

IV. GEOMETRY

A. Figure/Ground Discrimination

(Use Flip Book)

1. Star/moon/square: _____
2. Stars: _____

B. Basic Shapes

(Use Pattern Blocks)

1. "What is this shape called?"

a. (orange): _____

b. (green): _____

c. (blue): _____

d. (beige): _____

e. (yellow): _____

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C. Visual Memory of Geometric Design

(Use Pattern Blocks and Flip Book)

1. "I'm going to show you a design I made with these blocks. Then I'm going to cover my design. You are to make an exact copy of my design." (Allow multiple looks if necessary.)

a.

b. Tower 1 _____
c. Tower 2 _____
d. Or/Grn _____
e. Yel/Bg/Grn _____
f. Or/Grn Rocket _____
g. Diamond _____

Note:
Parts Approach _____
Configuration Approach _____
Use of Verbalization _____

IV. Geometry, cont'd.

2. "I'm not going to cover this design (3 diamonds). See if you can make it using blocks."

a. 3 diamonds

Note:
Block Chosen First _____
Building 2-D or 3-D _____
Non-reconcillation of color and perceptual cue _____

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D. Space Filling

(Use Pattern Blocks and Flip Book)

1. "Fill in this design." #1 _____
   "Another way?" _____ "Using all reds? _____

2. "Fill in this design." #2 _____ "Another way?" _____

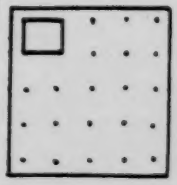
Note:

Builds with Smallest Pieces First _____
Starts on Edge (vs. Center) _____
For "Another Way" Substitutes an Equivalency for One Block _____

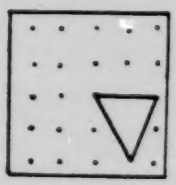
E. Geoboard Activities

(Use Two Geoboards)

1. a. "Make this same shape in the same position."
   b. "Name this shape." _____
   c. "Make a larger square." (Record response on this geoboard picture.)



2. a. "Make this same shape in the same position."
   b. "Name this shape." _____



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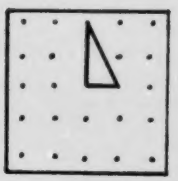
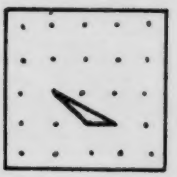
E. Geoboard Activities, cont'd.

3. "Copy this from the picture."

(Use Flip Book)





Note:

Use of Verbalization _____
Organizational Skills _____
Appreciation of General Design _____
Appreciation of Details _____
More Facility with Geoboard
Than With Blocks _____

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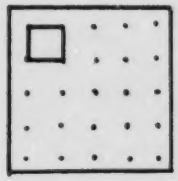
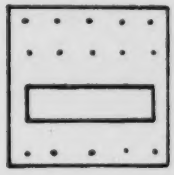
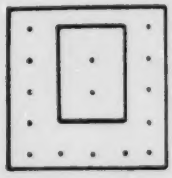
4. Intuitive Area Activities (Problem Solving and Learning Skills)

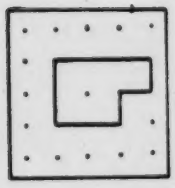
a. "One square covers this region."

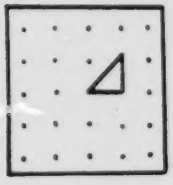


b. "How many squares would it take to cover each region?"

a.  ___

b.  ___

c.  ___

d.  ___

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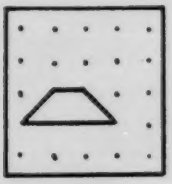
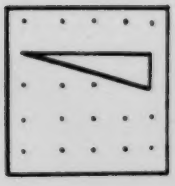
4. Intuitive Area Activities, continued

e.  ___

f. ___

g.  ___

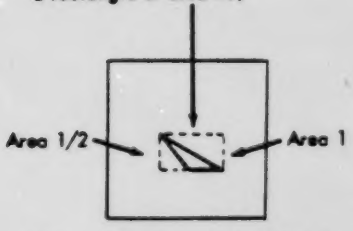
h. ___

Note Technique Used:

Cut and Piece Technique _____
Surround with a Rectangle and Find 1/2 the Region _____
(If necessary, teach this technique.)

Child able to learn and use the new technique. _____

(If necessary, give the child the hint of surrounding the figure with a rectangle of area 2.)



(Help point out separate regions with known areas.)

Note:

Child is able to integrate the partial areas to get solution. _____

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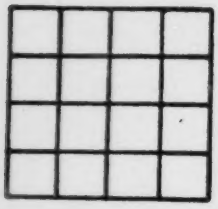
F. Problem Solving

(Use Flip Book)

1. 

"How many squares can you find?" _____
(The first answer is usually 16, 4 rows by 4 columns of little 1 x 1 squares.)

"There are more." _____
(The next one to be found is usually the large 4 x 4 square.)

"There are more." _____
(Give hints by showing one of the 2 x 2 squares.)

"There are more." _____
(If appropriate as a teaching exercise, see if the students can find all of the 2 x 2 squares, 9 of them, and all of the 3 x 3 squares, 4 of them. Success should be felt no matter how many of the 30 squares a student can find.)

2. Comparing Shapes

(Use Pattern Blocks)

a. Demonstrate:

[2 's → . Name of = 1/2.

"How many _____ → _____?" Name of _____.]

b. (1) 's → . Name of = _____.

(2) 's → . Name of _____ = _____.

(3) 's → . Name of = _____.

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Challenge Problem:

(4) (beige's) →

Name of = _____.

(There are many solutions to this problem by building matching shapes. For example:

is the same size and shape as

Hence 2 beige pieces equal 1 orange square.)

3. Three-Dimensional Figures

(Use Flip Book)
(Use cubes if necessary)

"How many cubes would it take to make each design?"

(If there is difficulty, ask child to build a few designs.)

1. Cube: 27 _____
2. 15 _____
3. 18 _____
4. 10 _____
5. 13 _____
6. 40 _____
7. 20 _____

4. Circle and Segment Problem

(Use Flip Book)

"The diameter of a circle is equal to 17 cm. Does a straight line touch or cross the circle, if the distance between the line and the center of the circle is:

6 cm. _____
8.5 cm. _____
10 cm. _____"

5. Region Problem

"Draw any four-sided figure so that one segment drawn through it will form four regions."

Hint: "Are squares, rectangles, and parallelograms the only four-sided figures?"

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V. Arithmetic Operations

(Use Flip Book)

A. "I don't care what you use to solve problems: fingers, chips, rods, drawing, writing or just your head. I will keep track of the answers for you."

4 + 1 =	6 - 2 =	2 x 3 =	22 x 4 =	18 ÷ 2 =
2 + 3 =	12 - 1 =	7 x 3 =	41 x 4 =	4)12 =
6 + 8 =	9 - 7 =	4 x 7 =	14 x 10 =	3)90 =
9 + 5 =	13 - 8 =	8 x 7 =	16 x 11 =	3)69 =
8 + 7 =	37 - 25 =	10 x 7 =	36 x 42 =	12)144 =
21 + 32 =	27 - 9 =	9 x 7 =		24)315 =
46 + 12 =	46 - 29 =	8 x 9 =		
32 + 5 =	101 - 16 =			
47 + 8 =	216 - 78 =			
35 + 16 =				
79 + 83 =				

(If an error is made performing arithmetic procedure mentally, ask the individual to write down how he 'worked the problem out.' Note discrepancy in mental vs. written procedure if any.)

Note:	+	-	X	÷
Recall of Facts				
Recall of Procedure				
Precision				
"Recipe Style"				
"Ball Park Answers"				
Compensatory Strategies				

B. Missing Addends:

8 + ☐ = 12 _____ ☐ + 3 = 9 _____

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C. Mental Arithmetic

1. "6 + 2 - 3 + 5 - 4" _____ (6)
2. "6 + 4 x 2 - 7 - 3 x 5 = " _____ (50)
3. "4 + 6 ÷ 5 + 2 - 1 x 3 = " _____ (9)
4. "100 ÷ 5 + 4 ÷ 8 - 3 = " _____ (0)

D. Addition Problem:

1. "Find the sum." 1, 3, 7, 9, + 2

"Can you find the sum in another way?" _____
(See if student uses the ten's method. _____)

2. Find the answer $\frac{250 + 250 + 250 + 250 + 250}{5}$

a. by direct computation _____
b. by pattern _____

E. Estimation Skills

(Use Flip Book)

"Look at each problem quickly. I shall cover it."

1. "Is the sum closer to: 1, 10, 100?" _____
"How do you know?" _____

2. "Is the product closer to: 30, 100, 500, 1000?" _____
"How do you know?" _____

3. "What is this product closest to?" _____

4. "What is this quotient closest to?" _____

VI. Arithmetic of Fractions and Decimals

A. (Use Fraction Bars.)
Spread out bars.

1. "Find a bar that shows 1/2." _____

2. "Find another bar equal to 1/2." _____
"What is its name?" _____

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B. (Use Tower of Bars)

1. "Color in a bar so it is worth 4/5." _____
   "Write the fraction numeral for 4/5." _____

2. "Color in another bar equal to your 4/5 bar." _____
   "Write its fraction numeral." _____

Note Technique Used:

Equivalent Fractions _____
Matching Lengths _____

3. "Write a fraction numeral for:

$\frac{6}{9}$ _____ $1\frac{1}{3}$ _____ $\frac{15}{12}$ _____

C. (Use Flip Book and Tower of Bars)

"Read these numerals. Color in the bars correctly."

	Reading	Coloring
$\frac{1}{2}$	_____	_____
$\frac{2}{3}$	_____	_____
$\frac{5}{12}$	_____	_____

D. Operations on Fractions

(Use Flip Book)

1. $\frac{1}{12} + \frac{5}{12} =$
2. $\frac{1}{2} + \frac{1}{3} =$
3. $\frac{4}{15} - \frac{1}{6} =$
4. $\frac{1}{4} \times \frac{1}{2} =$
5. $\frac{2}{3} \div \frac{4}{6} =$
6. $\frac{2}{7} \div \frac{14}{10} =$

24

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E. Decimals

1. "Write: .4 .16 1.5 .0023"

(Use Flip Book)

2. "Read:" .3 0.7 .02 1.6 1.005

F. Decimal Operations

1. .13 + .12 = _____ 2. .72 + .3 _____

3. .2 x .03 = _____ 4. .6)$\overline{.12}$

VII. Intuitive Problem Solving

(Use Flip Book)

A. Pictorial Graphs

1. Fruit

a. "Which fruit is most popular?" _____
b. "Which fruit is next?" _____
c. "How do you know?" _____

2. Families

a. "Which family has most people in it?" _____
b. "How do you know?" _____
c. "Which family has a grandmother?" _____
d. "How many sisters does Brown have?" _____

3. Jump Rope Contest

a. "Who jumped longest?" _____
b. "Who jumped the shortest?" _____
c. "Who jumped longer than Dodi?" _____
d. "How long did Beth jump?" _____
e. "How much more did Beth jump than Ada?" _____

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4. Map

a. "You are in the grocery store. You are going to the hardware store. Which way do you turn when you leave the grocery?" _____

b. "You are in the forest going to the library. Describe your route." _____

c. "You are giving directions to someone in the theater. They want to go to church the shortest way. What directions would you give?" _____

B. Applications

1. "Take 3 chips. Now take 4 more chips. How many chips do you have altogether?" _____

2. "How many chips do I have?" _____
Put the chips in an opaque container or bag.
"If I put 4 more chips in the bag, how many chips would there be altogether?" _____

Note Technique Used:

Match Two Sets _____
Counting on _____
Addition _____

3. "If I have 4 cars, how many wheels are there altogether?" _____

"If I have 6 kids, how many legs are there altogether?" _____

"If I have 6 kids, how many toes are there altogether?" _____

Note Technique Used:

Chips _____
Arithmetic _____
Drawing _____

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4. "Here are 13 chocolate chip cookies." (Give 13 chips.)
"Suppose we share them fairly among 3 kids. How many cookies would each kid receive?" _____

Note Technique Used:

One-by-One Method _____
Share by Groups _____
Arithmetic _____
Fraction Remainder _____

5. "If I buy 6 apples for 42¢, how much should I pay for 3 apples?" _____

Technique _____

6. "For every 30¢ Mary gives to charity, Fran gives 20¢.
If Fran gives $1.00 in all, how much does Mary give?" _____

Technique _____

7. Money (Optional)
(Use real coins and bills.)

a. "Name the money."

Show a penny _____ nickel _____ dime _____
quarter _____ dollar _____.

b. "Show me with money."

6¢ _____ 23¢ _____ 36¢ _____

(Note the coins used.)

c. "Suppose you bought some fruit for 72¢, and you had $1.00.
How much change would you get?" _____

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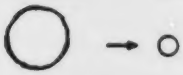
C. Analogies

(Use Flip Book)

1. _____
2.  _____
3. _____
4. _____
5. 3 → 18 _____
6. 5 → 8 _____

D. Sequences

(Use Chips)

1. R/B/R/B ..... _____
2. G/Y/R/G/Y/R ..... _____
3. B/Y/B/YY/B/YYY/B ..... _____

(Use Flip Book and Pattern Blocks)

4. Pattern Block Shapes _____
5. Hexagon/Red _____
6. △'s _____
7. Dice #1 _____
8. Dice #2 _____
9. _____
10. E _____

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11. _____

12. 3, 6, 9 . . . _____

13. 2, 1, 4, 2 . . . _____

14. 1, 1, 2, 3, 5 . . . _____

15. M T W T . . . . _____

E. Intuitive Functions

(Use Flip Book for Function Tables)

	Elements	Rule
a. $n \rightarrow 2n$	_____	_____
b. $n \rightarrow n-1$	_____	_____
c. $n \rightarrow 5n$	_____	_____
d. $n \rightarrow n^2$	_____	_____
e. $n \rightarrow 2^n$	_____	_____
f. $n \rightarrow 2n+1$	_____	_____
g. $n \rightarrow 2n+3$	_____	_____
h. $n \rightarrow n^2+n$	_____	_____

F. Word Problems

(Use Flip Book)
Have the student read the problem.

a. Cat/kittens _____

b. Susie/store _____

c. Bluejays _____

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G. Creature Cards

(Use Flip Book)

a. Gligs _____ e. Skints _____

b. Shlooms _____ f. Norleys _____

c. Gruffles _____ g. Quarks _____

d. Snorps _____

H. "If it takes 4 minutes to hard boil one egg, how long does it take to hard boil two eggs?" _____

VIII. Algebra

(Use Flip Book)

A. Integers

1. $^{+}2 + {}^{+}7 =$
2. $^{-}3 + {}^{+}5 =$
3. $^{-}2 + {}^{-}9 =$
4. $^{-}5 - {}^{+}3 =$
5. $5 - {}^{-}4 =$
6. $^{+}3 \times {}^{+}7 =$
7. $^{+}5 \times {}^{-}2 =$
8. $^{-}10 \times {}^{-}7 =$
9. $^{+}12 \div {}^{-}3 =$
10. $^{+}15 \div {}^{+}6 =$
11. $^{-}7 \div {}^{-}1 =$

B. Radicals

$\sqrt{144}$ $\qquad$ $8^{\frac{1}{3}}$

$\sqrt{-36}$ $\qquad$ $\sqrt{72}$

C. "Evaluate when a = 2; a = $^{-}5$:"

1.	a + 2	_____	_____
2.	2a + 1	_____	_____
3.	2a + 3 + 3a	_____	_____

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E. "Simplify if possible:"

1. $^{+}3a + {}^{-}5b$ _____
2. $-7x - {}^{-}2x$ _____
3. $3xy + 2x$ _____
4. $3(b + 2 - 3b)$ _____

F. "Factor:"

1. $15mn + 5m$ _____
2. $x^2 + 3x + 2$ _____
3. $12x^2 - 48$ _____

G. "Expand:"

1. $3(ab)$ _____
2. $2(x + 7)$ _____
3. $7(8 - 2y)$ _____
4. $3m\ (x + 3)$ _____
5. $(x + 1)(x + 4)$ _____
6. $(2 + x)(3 - x)$ _____
7. $(3 + y)(m - 7)$ _____

H. "Solve:"

1. $2x - 16 = 12$ _____
2. $3x + 5 = x + 2$ _____
3. $17 = 81 - 3x$ _____
4. $2(x + 2) + 5 = 4(x + 2) - (1 + 2x)$ _____
5. $x^2 - 5x = {}^{-}4$ _____
6. $x^2 + 17x - 12 = 0$ _____

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7. $\begin{cases} x + 3y = 7 \\ 2x - y = 8 \end{cases}$ _____

8. $\begin{cases} y = 7 + x \\ 3x - 1 = 18 - y \end{cases}$ _____

I. "Simplify:"

1. $\frac{1}{x} + \frac{1}{y}$

2. $\frac{2}{x+1} - \frac{3}{x+5}$

3. $\frac{1}{2x+4} + \frac{x}{x^2-4}$

J. Problem Solving

1. "...Which number cannot be 0?" _____
2. "...one more than 1/4..." _____
3. "... K = " _____
4. "...The time is..." _____
5. "...The area is..." _____
6. "...Traffic pattern..." _____

Learning Styles in Mathematics

In both teaching and clinical situations, definition of an individual's posture in mathematics includes not only a description of his level of achievement but also of the particular cognitive style with which he approaches mathematical situations. The definition of a preferred learning style for an individual is particularly important because appealing to this learning style and using the biases it implies maximize an individual's opportunity to learn mathematics.

In order to assess a learning style, an observer must watch carefully how an individual functions in a variety of mathematical settings. Both specific topics in mathematics (arithmetic, geometric, and logical) and the modalities in which they are presented (concrete, pictorial, abstract) must vary to insure the greatest opportunities to witness mathematics thinking in process. The characteristics displayed during particular mathematical situations as well as in the implementation of mathematical procedures deserve careful notation. The constellation of mathematical topics and procedures as well as the way in which they cluster generates a specific mathematics style.

In this study, an initial hypothesis of two learning styles was made which sorted mathematical skills and topics (as included in the MDPI) into two basic categories. Furthermore, the various processes observed as the activities were completed were sorted into these two categories. The categories reflected the characteristics that were mutually shared or required by the members of the categories.

LEARNING STYLE I

- Achievement
  - Better performance on addition and multiplication than on subtraction and division.
  - Procedures are adequate, but executed in a "recipe style."

- Answers not viewed through estimation techniques.

- "Math Sense" -- Problem Solving
  - Compensatory arithmetic strategies used in response to memory deficits indicate "one step at a time approach."
  - On sequences, better performance when numerical cues rather than perceptual cues are given.
  - On functions, possible to name pairs, but not general rule.
  - Creature Cards evoke slow approach, often characterized by verbal descriptions.

- Behaviors
  - Preference for a discrete model for number and place value.
  - Tendency to remember "parts" rather than "wholes" on visual memory exercises.
  - Performance better on geoboards than with shape blocks. (Geoboards lend themselves to verbal interpretation and cueing.)
  - Need for verbal rehearsal, reauditorization -- "talking through," "telling hands what to do."

- Deficits
  - Counting backwards.
  - Arrangements of components in visual memory exercises.
  - Seduced by external details of
    perceptual cues
    verbally presented math situations.

- Cannot select strategies in unstructured situations.
- Relies on external definition and directions.

LEARNING STYLE II

- Achievement
  - Good performance on counting backwards.
  - Subtraction and division concepts more facile than addition and multiplication.
  - Estimation techniques used.
  - Appropriateness of solutions good.
  - Impatience and imprecision with multi-step procedures.
- "Math Sense" -- Problem Solving
  - Compensatory arithmetic strategies used in reponse to memory deficits indicate large-scale pattern recognition.
  - On sequences, better performance when perceptually cued elements are given rather than numerical cues.
  - On analogy completion activities, responds well.
  - On functions, can recognize general pattern governing pairs, but may not be able to generate specific pairs.
  - Creature Cards evoke rapid recognition of elements of sets, yet errors in discriminating detail.

Purpose of Study: Description and Rationale, Cont.

- Behaviors
  - Preference for continuous model for number and place value.
  - Generates correct overall configuration in visual memory exercises.
  - Performs well on perceptual tasks, such as picturing geometric situations mentally or visualizing three-dimensional configurations.
- Deficits
  - Careless errors in answers where procedures are correct.
  - Impatience with detail.
  - May spontaneously come up with a correct answer but not know how it was obtained.

Careful consideration of the rating scales for subtopics of the MDPI, determination of the reliability of the subtopics, and the determination of test reliability were established before more detailed analyses of learning styles were attempted.

Mathematical Learning as it Relates to General Cognition

Of interest to the investigator were the apparent similarities of the characteristics associated with lateralized cerebral function and the characteristics associated with each hypothesized learning style in mathematics. Statistical correlations and analyses were made to study these possible similarities.

Comparisons of motor characteristics as evaluated in the neurological examination were made with the results of the MDPI. (In the clinical setting,

the neurologist "performs a complete neurological examination, including an extended examination for subtle classical abnormalities and developmental immaturity.")*

Furthermore, comparisons of general cognitive function as reflected in the neuropsychological findings were made with the findings of the MDPI through statistical analyses and techniques. (In the clinical setting, the neuropsychologist "performs selective cognitive tests and interprets the results as well as the qualities of the child's performance in terms of learning potential, specific areas of academic difficulty and learning style.")*

Teaching Strategies Related to Learning Styles in Mathematics

An important outcome of the definition of specific learning styles is the generation of more effective teaching techniques. If an individual is characterized as having a particular learning style in mathematics, then specific instructional strategies can be designed to be responsive to the elements unique to that style.

Once the character of the specific learning styles in mathematics was stabilized, then some specific differentiated approaches were designed. To generate a comprehensive listing of varied approaches to all of the key mathematical topics was beyond the scope of this study. However, typical examples were designated and are presented in Section V of this report ("Prescriptive Teaching") to highlight the tenor and techniaues required in making such differentiation.

Recommendations resulting from the diagnostic process extend beyond the scope of varied approaches to particular mathematical topics to more general classroom considerations. The following Overview of the Mathematics Diagnostic Process used at the Children's Hospital Medical Center, Boston, presents the general tenor of recommendations related to various types of observed behaviors.

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*Brochure, Learning Disabilities Clinic, Department of Neurology, The Children's Hospital Medical Center, Boston, Massachusetts.

OVERVIEW OF MATHEMATICS DIAGNOSTIC PROCESSES

Children's Hospital Medical Center, Boston, Massachusetts

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DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Level of Achievement in Mathematics (Which for most schools means computational skills and concepts)	• Strong for Age • Appropriate • Some Delay (one year or less) • Significant Delay (For a student of age >14, is the individual mathematically literate?)	Remediation: • In Class • Outside Class to Supplement Classwork • Outside Class for Total Math Program Support (in the form of): • Relaxed Time Requirements • Use of Manipulative Materials - to reduce memory - to provide conceptual framework - to provide applications • Encouragement ("Cheerleader") • Structuring ("Jiminy Cricket") • Calculators (Age >14)

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Recall Deficits	A. Recall of Facts (Use of compensatory strategies noted)	A. • De-emphasize automatic recall • Establish effective compensatory strategies within acceptable time limits • Relax time requirements, pressure, and constraints
	B. Recall of Multi-step Procedures	B. • Provide recipes • Strengthen conceptual foundations • Encourage estimation techniques
	C. Recall of Directions D. Recall of Mathematical Components (as seen in dictation and in problem solving activities)	C&D. • Make teacher and student aware of these tendencies • Repeat presentations • Highlight salient elements • Provide small group instruction to develop self-monitoring and compensatory strategies
	E. Visual Memory	E. • Minimize perceptual requirements • Minimize use of blackboard

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Learning Style in Mathematics	Learning Style I: • Preference for Verbal Cues • Deductive Processing • "Part-for-Part" Approach • Emphasis on Components Learning Style II: • Preference for Perceptual Cues • Inductive Processing • Emphasis on Gestalt	• Mathematical Models Geared to Preference • Type of Presentation Geared to Preference. • "At Risk" Topics, as a Reflection of Preference or Impairment Should Be Noted Note whether each style is: - Operative - Preferred - Impaired

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Establishment of Developmental Milestones in Mathematics	Some of the Milestones Noted Include: • Comfort with Symbolism (for example, no any specific types of reversal problems) • Motor Production • Conservation of Number • Conservation of Length • One-to-One Correspondence • Basic Notion of Quantity • Awareness of Basic Attributes (Size, Color, Shape, etc.) • Place Value (Grouping) Concepts • Concepts of Adding and Subtracting • Ability to Think Abstractly • Non-egocentric Approaches to Problem Solving	Highly Experiential Math Program Recommended When Milestones Are Not Yet Reached. "Push and Retreat" Strategy (Introduce formalism or specific mathematical interpretation; if it doesn't "take," then retreat until a later time in the maturational process.)

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIOR OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Strong "Math Sense"	• Cleverness in Thinking • Flexibility (Multiple and Varied Approaches) • Ability to See Patterns • "Curtailment"	Prediction of Success in Higher Level Mathematical Topics
Adequate "Math Sense"	• Ability to Select Appropriate Strategies from Repertoire of Familiar Ones • Ability to Extend Concept Beyond Immediate Application • Ability to Draw Inferences	
Weak "Math Sense"	• Ability to Apply Concepts Directly to Real Life Situations • Little Comfort with Abstraction or Complexity	

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Behavioral Interferences to Mathematical Progress	• Lack of Attentiveness - Extraneous Physical Activity - Distractability - Inability to Focus - Difficulty in Sustaining Behavior - Impulsivity • Problems in Organizing Work • Need for External Structuring In Order to Proceed, Especially with Iterative Processes • Perseveration • Sequencing Problems • Lack of Integration of Multi-step Procedures	Consideration of: • Piagetian Stages (Concrete-Pictorial-Abstract) • Role of Teacher • Number of Students in a Group • Procedures to Help Students Monitor Their Own Work

DIAGNOSTIC FORMULATION	TYPES OF BEHAVIORS OBSERVED	GENERAL TENOR OF RECOMMENDATIONS
Language Interferences to Mathematical Progress	Confusions in: • Directions • Applications • Arithmetical Algorithms (Procedures)	• Emphasis on Approaches Using Manipulative Materials. • Help in How to Read and Effectively Use the Format of Mathematical Textbooks, Descriptions of Mathematical Procedures and Word Problems.

THE INTEGRATION OF DIAGNOSTIC AND PRESCRIPTIVE CONSIDERATIONS THROUGH CASE STUDIES

Reviewing diagnostic considerations and then integrating them with prescriptive teaching approaches represent the concerted effort of neurology, psychology and mathematics. Yet the entire audience for the effort is the individual it serves--the individual who grows as a result. Since no two individuals are the same, general comments, although useful as they are, are never so vivid as when exemplified by real cases. To follow are selected case studies which are presented to help characterize how diagnostics and pedagogy can merge and mutually contribute to the understanding of one another.

Clinical analyses of the results of the administration of the Mathematics Diagnostic/Prescriptive Inventory over the past seven years show the MDPI to be an instrument sensitive to a wide variety of cognitive styles and characteristics. Some of the conclusions which emerge refer to general learning issues. Others deal more particularly with the learning of mathematical topics and procedures. After watching an examinee pursue the various activities in the MDPI, the examiner is left with many valuable intuitions and insights.

These particular cases were chosen so that some of the more frequently encountered profiles in mathematics may be highlighted. The individuals tested represent a cross section of various cognitive styles:

- individuals with a preference for Mathematics Learning Style I
- individuals with a preference for Mathematics Learning Style II
- individuals whose impairments are in their preferred Learning Style
- individuals for whom the preferred Mathematics Learning Style is not easily discerned
- individuals for whom work with manipulative materials at a purely concrete stage is too unstructured

The impairments described are varied and are typical of those often encountered.

They include

- questionable developmental maturity in mathematics
- specific learning disabilities in mathematics in the presence of limited cognitive potential
- specific deficits in mathematics in the presence of a strong "math sense"
- organizational problems which hinder mathematical achievement
- problems with the production of responses
- specific memory deficits
- problems with the integration of the two Mathematics Learning Styles
- composites of problems which may or may not put the individual "at risk"

These case studies are not offered as a comprehensive view of learning disabilities in mathematics, nor should they be considered authoritative in the impressions and recommendations given. Rather, they are merely suggestions of possible interpretations of performance, possible relationships between findings, and possible reporting styles. Although the names of the individuals have been changed to protect the confidentiality of the reports, the discussions reflect actual cases.

The following chart summarizes the case studies presented, as well as the general character of the mathematical profile for each individual.

Individual	Age	Grade	Level of General Ability	Preferred Learning Style for Mathematics	Apparent Concerns
Jennifer	6-11	1	Not apparent	No preference clear	Developmental concern
Aaron	7-11	2	Bright	Strong Math Learning Style I and Learning Style II	Organizational deficits
Jay	10-5	4	Average	Math Learning Style I	Organization deficits, Imprecision, Problems with preferred Learning Style. "At risk."
Irwin	11-2	5	Average	No preference clear	Integration deficits and emotional concerns
Ellen	11-6	3	Slow	Math Learning Style I	Generally slow progress
Adam	11-10	*	Bright	Math Learning Style II	Language impairment
Alex	11-11	6	Bright	Math Learning Style II	Production deficits, Organizational deficits
Julia	13-2	8	Average	Math Learning Style II	Math Learning Style I
Michael	14-4	9	Slow average	Math Learning Style I	Organizational deficits
John	16-2	10	Average	Math Learning Style I	Math Learning Style II
Paul	18	12	Bright	Math Learning Style II	Lack of confidence, Math Learning Style I

(* Ungraded school)

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Jennifer
D.O.B.: 4/15/71
C.A.: 6-11
Date of Evaluation: 3/23/78
Grade: 1

Jennifer was interviewed to assess her achievement, ability, and learning style in mathematics. This interview was part of a total evaluation whose focus was on appropriate academic planning. School reports note that Jennifer has writing problems, "incomplete knowledge of numbers" and attentional problems.

Previous Testing:

Key Math 10/77 G.E. 0.8

DISCUSSION:

Oral Counting: Deficiencies Apparent

Jennifer could only count up to 30. After that, she counted by tens. Within these limits, Jennifer could count on. Jennifer could not count backwards. She could count by 5's and 10's up to, but not beyond 100, suggesting only rote control of the sequences.

Basic Quantitative Concepts: Deficiencies Apparent

Jennifer demonstrated a basic concept of number -- the cardinality of a set. However, her actual counts were often inaccurate. Some intuitions for place value were apparent. There was no preference for a number model (discrete sets versus continuous quantities) indicated, since her level of development is still very basic and precludes such assessment.

Basic Quantitative Concepts, cont'd.

The concept of conservation of number did not seem well developed, suggesting that a significant arithmetic milestone has not been reached yet. Place value concepts were unfamiliar to Jennifer.

Numerical Symbolism: Deficiencies Apparent

Jennifer could write numerals up to 12 and read numerals through 15. No reversals of symbols or digits were apparent.

Arithmetic of Whole Numbers: Serious Deficiencies

Jennifer could only generate addition facts by using her fingers. However, she did not know how to proceed if one addend were greater than 5 since both hands were necessary to represent the first addend, leaving her at a loss for representing the second addend.

Jennifer's subtraction concepts did not seem well established. She could not deal with simple one-digit problems.

Informal Geometry: Deficiencies Apparent

Jennifer had no difficulty with figure/ground discrimination or in recognizing basic shapes. However, her memory for geometric configurations was quite deficient.

Jennifer had difficulty with even the simplest designs. She had difficulty both in remembering all the component parts as well as organizing them. A significant factor seemed to be the number of elements. With repeated cuing, she eventually could deal with three to four elements in a simple design. Greater complexity, even with additional cuing, seemed to overwhelm her.

Intuitive Problem Solving: Basic Level of Performance

Jennifer could not perform well on any of the informal problem situations. She seemed somewhat egocentric in relating situations to herself. Any degree of complexity confused her.

Jennifer could discern patterns in simple sequences. Any other puzzles seemed without relevance to her. She seemed to be working securely only at the concrete direct levels.

SAMPLE CASE STUDY
Jennifer, Page 3

BEHAVIORAL ASPECTS:

Jennifer was friendly and cooperative throughout the interview. Her responses were often delivered deliberately. She needed encouragement and careful cuing in order to proceed. She elicited patient and careful presentations and support from the examiner.

IMPRESSION:

Jennifer's achievement is seriously below grade level, with specific deficits in numerals, addition, subtraction and place value. Her performance was more like that of a kindergarten student, with readiness skills lacking.

Jennifer's developmental milestones in mathematics seemed generally appropriate. It is not unusual that conservation is not well established for children less than eight years old. Of some concern, however, is her lack of appreciation of grouping (place value) concepts.

It is difficult to assess Jennifer's basic "math sense" since her strategies seem immature. The lack of maturation in this area is characterized by inability to focus, inability to discern key aspects to a situation, and a need for structure and directness.

Jennifer's recall skills might be suspect on the basis of the present performance. Jennifer has developed no recall of arithmetic facts and has poor visual memory. However, other aspects of recall can be assessed only after further developmental maturity.

RECOMMENDATIONS:

A specific remediation program is warranted by Jennifer's current level of achievement in arithmetic and in her particular learning style.

The instructional program should emphasize an experiential approach, from which mathematical concepts might be developed. The Piagetian approach to concepts through a careful sequence of activities at the concrete, pictorial and then abstract level seems particularly appropriate. The most important aspect of the program should be the careful integration of these various experiences for each concept.

Jennifer's progress should be carefully monitored to assess whether appropriate developmental milestones in mathematics are achieved, particularly conservation of number. Without the establishment of conservation of number future arithmetic concepts and skills cannot proceed smoothly.

Jennifer's learning style is marked by all the characteristics generally associated with younger children. She has difficulty focusing on non-personally directed situations, dealing with multiple stimuli and considering concepts through highly perceptual applications.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Aaron
D.O.B.: 3/9/70
C.A.: 7 - 11
Date of Evaulation: 2/2/78
Grade: 2

Aaron was interviewed to assess his achievement, ability, and learning style. This interview was part of a total evaluation whose focus was on appropriate academic planning. School reports indicate that the major concern is the production of his work. Aaron is said to have difficulty with attention and the completion of tasks.

Previous testing: None

DISCUSSION:

Oral Counting: Skills At or Above Grade Level.

Aaron could complete all tasks well.

Basic Quantitative Concepts: Appropriate Responses.

Numerical Symbolism: Significant Deficits.

Aaron could read and copy all numerals appropriately. When writing numerals, he consistently reversed the symbol for each digit (see attached). It is noteworthy that the place of the digit was always correct; the positions of the digits were not reversed. Aaron also demonstrated an unusual technique of writing numerals; he started at the bottom-most point and proceeded upwards.

Arithmetic of Whole Numbers: Appropriate Responses

Aaron's performance on all arithmetic task suggested that addition, subtraction, and even multiplication concepts are well established.

Arithmetic of Whole Numbers, cont'd.

Aaron demonstrated automatic recall of all addition and subtraction facts. In solving one-digit multiplication problems, Aaron used pictorial representations (slashes) to help him arrive at responses.

Aaron could apply the two-digit addition and subtraction algorithms both with and without regrouping. Again, Aaron was handicapped by imprecision. He clearly knew the correct general procedures. However, in implementing these procedures he used incorrect terms, showed difficulties with subroutines and a lack of attention to operation signs.

Informal Geometry: Area of Relative Strength

Aaron demonstrated appropriate recognition and identification of basic shapes.

In the activities requiring visual memory of complex designs, Aaron seemed able to note the components of the configuration as well as to appreciate the overall design. The errors which Aaron made were characterized by omissions, reflections and lack of general organizational skills.

In the activities on the geoboard, Aaron responded well. The inherent organizational cues provided by the geoboard seemed to help Aaron. Nonetheless, precision and organizational difficulties were not completely overcome.

Despite these deficits, Aaron demonstrated excellent appreciation of components as well as overall arrangements. His ease and enthusiasm for dealing with geometric situations were noteworthy.

Intuitive Problem Solving: Area of Strength

Aaron demonstrated very nice analytic strategies in discerning governing patterns on sets. He could apply math concepts to real life applications. Aaron also did quite well on the activities involving reading and interpreting pictorial graphs.

Behavioral Aspects:

Aaron was serious and hardworking throughout the interview. He was unusually persistent in pursuing an activity. Aaron did not seem able to leave a problem incomplete. He seemed to feel he must complete it successfully.

SAMPLE CASE STUDY
Aaron, page 3

IMPRESSION:

Aaron's achievement in mathematics seems quite soundly established. In fact, he demonstrates facility, understanding and skills that are not usually found in second graders.

Aaron's overall learning style seems the most salient aspect of his performance. All activities were characterized with organizational and precision difficulties. With written numerals, geometric configurations and multi-step procedures, Aaron demonstrated difficulties orienting figures, including all salient components, and integrating sequential processes. Aaron often used verbalization to help organize his strategies. Unfortunately, these verbal cues themselves were imprecise. However, these "imperfections" did not seem to reduce the help which the verbalizations provided.

Aaron's overall "math sense" seems quite strong, with appropriate intuitions and logical thinking evident.

Aaron's intensity and diligence may be of some concern, since their degree is disproportionate to his level of achievement.

Aaron demonstrates significant difficulty in numerical symbolism. Since the errors are consistent and involve only symbols and not positions, this problem of symbolism need not make further mathematical topics inaccessible.

RECOMMENDATIONS:

On the basis of the concepts and skills he demonstrated, Aaron has learned quite well from the present instructional program. Aaron should be made aware of the high level of his accomplishments.

The problem with symbolism can be easily circumvented by teachers learning to interpret Aaron's particular versions of numbers, providing a tape recorder for him to talk to, or providing a hundreds chart or other device on which he can merely circle the correct answers.

Since Aaron has no difficulty copying or interpreting numerals presented to him, instructional approaches need not be modified.

The production of numerals is discouraging to Aaron, since it requires so much effort and yields so few positive results. Recording of problems should be kept at a minimum.

By building Aaron's confidence in his math ability and in himself, in general, the learning experience might become more relaxed, and some of the more uncomfortable behavioral characteristics previously noted may be moderated.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Jay
D.O.B.: 11/11/68
C.A. 10-5
Date of Evaluation: 5/3/79
Grade: 4

Jay was interviewed to assess his achievement, ability, and learning style in mathematics. Jay is reported to have a history of learning problems.

Previous Testing: None

DISCUSSION:

Oral Counting: Appropriate Responses

Jay responded rapidly and accurately in producing all counting sequences.

Basic Quantitative Concepts: Appropriate Responses

Jay seems to have acquired all basic number concepts. In fact, Jay emphasized place value concepts spontaneously. Jay seemed to prefer a discrete (chip) model for number over the continuous (rod) model. Although Jay could interpret number and place value concepts with rods, his responses were less elegant. Interestingly, he spontaneously placed rods horizontally, end-to-end, even though a vertical arrangement was a more useful one.

Numerical Symbolism: Appropriate Responses

Jay could read and write numerals into the tens of thousands. In the dictated arithmetic problems Jay's recall of elements was adequate. However, in complex situations, he lost details. His organization of elements was not well established.

Numerical Symbolism, cont'd.

Jay frequently asked for repetitions of stimuli and then proceeded to reauditorize them himself.

Arithmetic of Whole Numbers: Deficiencies Apparent

Jay's recall of arithmetic facts did not seem automatic. This could be deduced from the deliberateness of his responses in addition and subtraction. However, he demonstrated no apparent compensatory strategies. By way of contrast, in producing multiplication facts, he could not produce the solutions without building from known facts (a nice compensatory strategy).

Jay's recall of procedures seemed adequate. He could apply the addition and subtraction algorithms both with and without regrouping. He could apply the multiplication algorithm when the multiplier was a one-digit numeral, but not a two-digit multiplier.

Interestingly, Jay seemed to perseverate the division procedure through addition, subtraction and multiplication examples. However, when division problems were presented, he said he hadn't had division before.

These arithmetic situations seemed painful for Jay. All responses were deliberate. Even so, imprecision marked complex problems. Jay seemed to want to respond rapidly and seemed frustrated by his need for deliberateness.

Jay's overall performance is more like that of a fourth grader. Qualitatively, these are concerns based on his deliberateness, imprecision, and almost impulsive style.

Informal Geometry: Deficiencies Apparent

Initially Jay had difficulty with the figure/ground discrimination problems. He was seduced by the internal details. However, he self-monitored and self-corrected to produce the correct response.

In the activities involving visual memory of complex designs, Jay generally recalled the elements and approached the designs through their components. However, Jay had consistent organizational problems, marked by rotations and reflections.

In the activities on the geoboard, Jay responded well. This often happens, since the geoboard offers inherent organizational cues. In the problem solving situation, Jay

Informal Geometry, cont'd.

formulated highly segmented strategies. When iterative procedures were required, Jay seemed to lack persistence.

Intuitive Problem Solving: Adequate Responses

Jay did his best work when situations were direct and stimuli were simple. In more complex situations, stylistic and behavioral concerns seemed to interfere with his strategies and responses.

Jay demonstrated persistent problems with highly verbal situations and with complex directions. His responses in arithmetic applications were particularly deliberate, since the problem situations were presented orally.

Jay could deal with activities discerning patterns in sequences, analogies and sets of elements, when the cues were simple. When multiple cues were to be assimilated, Jay had difficulty and produced rapid responses, without really thinking about their quality.

Behavioral Aspects:

Jay was friendly and cooperative throughout the entire interview. Answering with speed seemed to be the source of many problems for Jay. Jay has placed a high premium on producing rapid responses despite his apparent style and need for deliberation. When he himself sensed his own deliberateness, he became frustrated and impatient. When actually producing quick responses, Jay was often imprecise and inaccurate. On an encouraging note, Jay has developed some self-monitoring skills and spontaneously self-corrects some of his initially incorrect responses.

Unfortunately, these monitoring strategies do not seem securely established. They also seem to depend on Jay's current degree of discouragement and persistence.

IMPRESSION:

Jay's achievement in arithmetic is below grade level with specific deficits in multiplication procedures, division concepts, division procedures and fraction concepts.

Jay demonstrated selected memory deficits with mathematical contexts: recall of arithmetic facts and recall of directions. Jay's recall of mathematical components

Impression, cont'd.

of situations and visual memory were adequate. Jay's recall and application of multi-step procedures were very encouraging.

Stylistically, Jay seemed to prefer stimuli and approaches that emphasized careful, step-by-step procedures. He seemed to respond best to verbal stimuli and often reinterpreted situations orally for himself through reauditorization. Unfortunately, Jay demonstrated real difficulty in organizing the components of situations. Jay's apparent tendencies to evoke perseveration should also be noted.

Behaviorally, Jay demonstrated some characteristics that might interfere with learning and the quality of his responses. Notably, Jay's concern for speed conflicts with his inherent need for deliberation. Furthermore, Jay's tendency toward discouragement and impatience often disrupted his approaches.

RECOMMENDATIONS:

Although Jay's performance in arithmetic is only one year below grade level, his particular learning style and behavioral characteristics put him "at risk" for his future work in mathematics.

Therefore, both a remediative and supportive instructional program seem warranted to improve skills and to prevent further deterioration of achievement. Of particular concern will be establishing monitoring and organizational skills. Initially those skills will need to be provided externally as future mathematics topics are inherently more complex, marked by multiple cues and multi-step procedures. Eventually, the internalization of these skills should be encouraged.

Children with recall deficits and a learning style similar to Jay's often benefit from instructional approaches that emphasize the development of mathematical concepts and procedures through the manipulation of concrete materials. Careful transitions can then be made from conceptual aspects of the activity at the concrete level to a more precise recording of the activity with abstract, numerical symbols. Not only do such approaches emphasize the application of concepts, but they offer models which may serve as alternatives to abstract memory for the specific components of a concept or procedure.

The choice of mathematical model is especially important in view of Jay's preferred learning style. A model which emphasizes discrete sets in which analysis elicited in a highly deductive component-by-component form would seem most appropriate.

Recommendations, cont'd.

Usually, these models lend themselves to careful verbal interpretation.

Jay's learning style also suggests the need for instructional strategies which are segmented into meaningful, smaller components which are carefully sequenced.

In response to Jay's memory deficits, the recall of arithmetic facts might be de-emphasized. Compensatory strategies for generating facts (counting on, counting backwards, generating unknown facts from known facts, the use of only one set to cue arithmetic computation, etc.) might be substituted.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Irwin
D.O.B.: 2/14/68
C.A.: 11-2
Date of Evaluation: 4/19/79
Grade: 5

Irwin was interviewed to assess his achievement, ability, and learning style in mathematics. He was referred for this evaluation by his parents who expressed concern about his current school progress and the desire for help in planning a viable junior and senior high school program for him.

DISCUSSION:

Oral Counting: Good Strengths

Irwin performed all forward counting, counting on, and counting backwards tasks with speed and accuracy. His voice kept a steady pace as he moved from one decade to another, and he did not resort to "sing-song" rhythms. He could count by 10's, by 5's, and by 2's, and had no problem going beyond 100 in any of these situations.

Numerical Symbolism: Appropriate Responses

Irwin was able to write numerals correctly, showing neither reversals of digits nor place value. His ability to remember what number was said and to write it down correctly was good. He was able to record addition and subtraction operations dictated orally. He used correct symbolism in both vertical and horizontal notations. His writing was neat, and he took care of his own accord with the details of putting in the commas and in lining up the place value columns. He was also able to read numerals correctly well into the ten thousands.

Basic Quantitative Concepts: Appropriate Responses

Basic concepts concerning cardinality, ordinality, and comparison of numbers seemed

well established. In counting collections of dots in a variety of configurations, he had no trouble with one-to-one correspondence, nor did he ever forget his starting place no matter how randomly the collection was pictured. He also took care to double check his answers. With encouragement, he was able to check the 5 by 5 array of dots by multiplying, after having solved and checked it by means of counting techniques.

In the activities involving number, he functioned well with both the "chips model" (discrete view of number) and the "powers of ten blocks" (continuous view of number). The understanding of place value (ones, tens, hundreds, thousands) in both of these models showed good insight and training. The preference for one model over the other (which frequently shows in some students) did not seem pronounced in Irwin's work. He is fortunate in that he has the potential to build on either of these two firm foundations.

Arithmetic of Whole Numbers: Some Deficits in Multiplication and Division

Irwin has memorized his addition facts so well that he was able to do one-digit and two-digit addition rapidly in his head, even when pencil and paper were allowed. On more difficult addition problems involving several regroupings ("carrying"), he was able to obtain correct answers by careful written work.

His subtraction techniques were accurate, yet less facile than his addition work (a common situation). He was able to apply the subtraction algorithm when there were both regrouping ("borrowing") and no regrouping required.

His one-digit multiplication facts have been well memorized, and he was able to do some simple one-digit times two-digit problems in his head. Yet his ability to multiply two, two-digit numbers on paper showed inaccuracies in remembering to "carry," confusions when zeros were involved, and a lack of awareness of patterns for multiplying by 10 and 100. He needs work in these areas.

He was able to divide correctly using the division algorithm. However, he does not know patterns for dividing by 10. He also needs work in rounding and estimating so that he does not need to write so many multiplication problems off to the side. These deficiencies in division are not critical as he will have much more formal work in division in the sixth grade.

Arithmetic of Fractions and Decimals: At Introductory Level, Some Deficits Apparent

At present, Irwin's understanding of fractions is minimal. He was able to name fractions correctly when shown pictorial representations. However, he was not

clear about the basic meaning of a fraction nor about the ways to obtain equivalent fractions. The operations with fractions also showed confusion and an attempt to apply "recipes" incorrectly and without understanding. For example, he consistently added two fractions by adding numerators and adding denominators without his knowing that he had the wrong process (even when estimation techniques would have indicated the illogical nature of his answers). These misconceptions are not surprising at his age, as fractions are not emphasized in Grade 5. However, when they are covered more carefully in Grades 6 and 7, Irwin would benefit from having the operations approached through concrete models such as Fraction Bars or Fraction Tiles so that the algorithms can take on tangible meanings. His overall performance in this evaluation indicates that this is a topic which may be difficult for him and one for which he may need extra time and help to master.

The topic of decimals also shows minimal understanding and exposure. Irwin reads decimal notation such as .3 as "point 3" rather than "three tenths," and .12 as "point one two" rather than "twelve hundredths." He does not link decimal notation either to fractions or to place values. He is not clear about lining up decimal points when adding two decimals. His good performance with whole numbers indicates that with persistence, decimals will be accessible to him. Within the next two years, he should be taught estimation techniques and how to use a hand-held calculator to remove some of the stress when problem-solving in his more advanced mathematics work. Irwin seems to function best when he's not coping with too many variables at once.

Informal Geometry: "Parts" Approach

In looking at a complex line drawing and having to draw his own replica of it, Irwin proceeded systematically for a nearly perfect performance.

In the activities requiring visual memory, he was able to use blocks to build accurately the picture he had viewed. He seemed to be approaching each configuration through its component parts.

On the other hand, he showed difficulty in the reverse setting — taking geometric shapes and putting them together to make a whole design where the pieces were not pictured. He seems to be very reliant on internal detail.

In fact, in solving the Creature Cards, he was so caught by detail that he could not perform the task of classifying into larger categories. He could not see that things could be alike or different in global ways. This extreme "literal interpretation" of Irwin's learning style came out in other aspects of the evaluation as well.

Intuitive Problem Solving: Mixed Performance

Problem solving tasks were presented in a variety of situations. The goal in these activities is to assess the individual's "math sense" as seen in the ability to distill key information, the ability to formulate approaches to problems, the ability to apply and pursue reasonable strategies, and the ability to apply logical thinking. The need for computational skills to find solutions is minimized.

Irwin's performance was strongest when situations were clearly defined. For example, he was able to read a graph when pictorial data were given, yet not when one block equal d 5 rather than 1. He tended to want to rely more on memory than on his own ability to figure things out on the spot. He seemed to feel that he had to answer fast, to answer without writing anything down, and even to anticipate the question — all factors which seemed to put added stress on him and to stamp out any chance of his utilizing his best potential for reasoning.

He seemed to perform equally on arithmetic versus geometric problems, but the two abilities didn't seem to work together for him on problems which could have been approached either way.

This evidence and other aspects of the evaluation seem to indicate less of an issue of hemispheric lateralization in Irwin than in many children (although the left hemisphere seems to be his preferred one), but more of a question of the maturity of his "corpus callosum," the area of communication between the two hemispheres. It should be noted that the "corpus callosum" generally matures between the ages of 8 to 12, and does not seem to be fully matured in Irwin's case. He seems to lack the internal communication in the thought processes required in multi-step thinking. It would be interesting to follow changes in his learning capacities in the next two to three years of development. The evaluation does not give indication that he should be classified as "learning disabled."

IMPRESSION:

The level of Irwin's performance in mathematics seems appropriate. However, several salient characteristics emerged. Critical to Irwin's progress in school is his self-concept. He needs an abundance of positive reinforcement, but honestly given. He knows when he is performing well and takes great pride in his good work. He also knows when he is not performing well and no matter how much one might try to convince him that the task may have been unreasonable, he feels failure deeply. He does not bounce back easily from even the slightest dent. Stressful situations mean that he performs less well than his true capacity. After experiencing difficulties in certain sections of this evaluation, he "didn't know" things that he "had known" five minutes earlier.

His level of sensitivity and desire to do well are so acute that he is "reading" every look and cue from the person in authority.

His stressful times can be "read back" very easily -- he swings his leg, twists his foot, sits up in his chair, rubs the side of his pants, answers impulsively without thinking first, and talks in a "breathless voice."

He seems to have the view that he ought to be perfect. He expressed great concern over one of his model cars that was less than perfect. That particular one was far more significant to him to talk about than the numerous "beautiful" models that were complete.

It is important for him to realize the significance of a two-year age difference between himself and his older brother. Otherwise, he will always feel defeated before he starts any new task. Perhaps there are some activities that he could get to do "first" or exclusively. He seems very responsible -- for instance, he put the extra chair back that we were using immediately when we finished, and he took care to put the caps back on the pens without hesitation.

When Irwin is having success, he has an extremely good attention span. He is able to sit for a long time without getting restless. He is the type of student who would benefit from a "big brother" type of tutor who would move him ahead in his work in a supportive but playful way. He needs to see humor in situations and feel less need to interpret things so literally. He reads so well that perhaps some well chosen stories involving boys his age might give him other models.

RECOMMENDATIONS:

Irwin needs to be taught in small classes with a caring teacher. He needs to be challenged in a supportive way, to have immediate feedback on his work, and to make steady progress. He should not be in too competitive a situation where he is apt to lose confidence in his ability.

It may well be that a carefully chosen private school would be a good choice for Irwin. The size and diversity of programs in his present public school setting, particularly at the junior high level may be overwhelming for him. Structured, small classes would help him build on his strengths. Consideration might be given to having him enter the fifth grade next year rather than the sixth to maximize his chances of success and to give him time to grow physically. However, the shift in friends and the fact that his present school seems like a constructive place for him might mean that the change should take place following the sixth grade. Irwin is at an age where his learning capacities and physical growth are changing rapidly. I advise a follow-up evaluation in a year's time.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Ellen
D.O.B.: 8/13/66
C.A.: 11-6
Date of Evaluation: 3/9/78
Grade: 3

Ellen was interviewed to assess her achievement, ability, and learning style in mathematics. Ellen is presently enrolled in the third grade. School reports indicate a history of academic difficulties. This evaluation serves to aid in planning academic goals for Ellen.

Previous Testing: None

DISCUSSION:

Oral Counting: Some Deficiencies

Ellen had difficulty counting by 5's and 10's when asked to continue beyond 100. This would suggest she has mastered a rote sequence but does not understand the way it was generated. Ellen could not count by 2's. However, with help she was able to skip count every other number.

Basic Quantitative Concepts: Some Deficiencies

The basic number concepts seemed well developed. However, the intuitions regarding the use of place value did not seem established. Ellen had real difficulty understanding that one object could stand for many single elements. She seemed insistent on seeing every number as a set of ones.

Ellen seemed facile with neither discrete (set) nor continuous (rod) models for number. Her ability to learn new models and interpretations seemed limited. This difficulty with using models suggests rote numerical skills and quantitative concepts rather than an application or understanding of them.

Numerical Symbolism: Deficiencies Apparent

Ellen appropriately read and wrote numerals up to the thousands. The mechanical process of writing the numerals, although accurate, was laborious and time consuming. Ellen also had difficulty in organizing the components of dictated arithmetic problems.

Arithmetic of Whole Numbers: Significant Deficiencies

Ellen seemed to have mastered most aspects of the addition procedure. Her recall of facts and application of algorithms were appropriate. It should be noted that Ellen used reauditorization to cue herself, by reminding herself of the steps of the procedure she used.

Ellen did not demonstrate the same proficiency in subtraction. The underlying concept was established. She did not recall subtraction facts and seemed to need to use chips to find a solution. In performing two-digit subtraction, she could subtract each column, but could not remember the components well enough to arrive at an answer. Integrating the partial differences into a meaningful solution was particularly difficult for Ellen.

Ellen demonstrated no familiarity with the concepts of multiplication, division nor fractions.

Ellen's overall achievement in arithmetic was more like that of a second grader than a third grader.

Informal Geometry: Adequate Performance

Ellen had difficulty with visual memory involving complex configurations. However, she was able to construct simpler designs. Ellen's problem solving skills in the geometric context seemed somewhat limited. She could only deal with simple and direct situations.

Intuitive Problem Solving: Basic Level of Performance

Ellen's responses on the various problem solving activities were more like that of a younger child. She was tantalized by the numbers in the applications section and ignored the context in which they were found. Ellen's interpretation of graphing situations was limited and her analysis was direct.

Ellen had great difficulty in discerning patterns between pairs or within sequences of numbers or shapes. She found it difficult to formulate a strategy of attack or to identify salient cues.

Behavioral Aspects, cont'd.

Ellen was interested, cooperative, and hard working throughout the interview. She demonstrated some difficulty with directions, often requesting them to be repeated or clarified. Ellen seemed to need reauditorization to cue herself to proceed.

Ellen seemed very deliberate in learning new concepts or skills. However, skills she had learned seemed well established.

IMPRESSIONS:

Ellen's achievement in arithmetic is significantly below grade level and extremely deficient with respect to age level. Specific deficits include place value, subtraction, multiplication, and fractional concepts.

Ellen's general "math sense" seems to be at a very basic level. Ellen responds best when situations and problems are direct and simply presented. She is easily diverted by multiple stimuli. Her ability to formulate and pursue strategies within arithmetic, geometric, or logical contexts seems quite limited.

Ellen's memory of well-practiced routines and visual memory seemed intact. However, her recall of directions, components of mathematical situations and multi-step procedures seems deficient.

Ellen's particular learning style, characterized by the need for very simple instructions, reauditorization, motor problems in recording and the need for extended time allotments, seemed to be salient aspects to her performance.

RECOMMENDATIONS:

In view of Ellen's academic history and her relative lack of success in regular learning situations, a substantially separate environment for learning mathematics is warranted. Ellen's ability and learning style mandate very special considerations.

If Ellen's achievement does not improve significantly in the future, modification of traditional academic goals in mathematics is indicated.

Ellen's instructional program should emphasize simple sequences of component tasks. Carefully presented and repeated, minimal alternatives should be presented, offering instead the single most practical approach.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Adam
D.O.B.: 4/8/66
C.A.: 11-10
Date of Evaluation: 2/2/78
Grade: Non-Graded School Setting

Adam was interviewed to assess his achievement, ability, and learning style in mathematics. This interview was a part of a total evaluation whose focus was on appropriate academic planning. Adam has had a history of language problems and concomitant academic difficulties. His present mathematics achievement is described by the school as on a grade three to grade four level, with particular note made of his tendency to make careless errors.

Previous Testing: None

DISCUSSION:

Numerical Symbolism: Appropriate Responses.

Basic Quantitative Concepts: Appropriate Responses.

Oral Counting: Appropriate Responses.

Arithmetic of Whole Numbers: Significant Deficiencies

Adam's recall of addition and subtraction facts seemed quite good. He correctly applied the addition and subtraction algorithms both with and without regrouping.

Occasionally, Adam made impulsive first and even second responses. However, he generally could find the correct answer with prodding. It is unclear whether or not he can monitor this impulsivity of response on his own.

SAMPLE CASE STUDY
Adam, Page 2

Arithmetic of Whole Numbers, cont'd.

Adam understands the multiplication concept. He can generate multiplication facts, but his recall of them does not seem stable. Adam demonstrates only primitive understanding of the multiplication algorithm. Multiplication seems to be the topic in which he is now working.

Adam demonstrated basic understanding of division, the fraction numeral and the concept of a fraction.

Adam's problem solving ability in the numerical context seemed quite good. He could recognize patterns and short cuts.

Informal Geometry: Some Deficits Apparent

Adam's figure/ground discrimination seemed appropriate. He could adequately perform tasks that required visual memory of geometric configurations if the number of elements were small -- less than three. However, when more elements were involved, Adam required repeated trials in order to remember and assemble all the components. In these activities, Adam clearly preferred a component approach to configurations.

Adam's performance of the geometric problem solving was strikingly good. The activities were portrayed on a geoboard. Adam did not seem to have difficulty within this mode.

Intuitive Problem Solving: Significant Deficits

Adam's performance on the applications and logical puzzles was especially poor. However, most of these activities are couched in less than simple language. It is unclear if Adam's intuitions and logical strategies were primitive or if language considerations prevented a reasonable attack on the activity.

Behavioral Aspects:

Adam was friendly and engaging during the enitre interview. He often had difficulty with directions. He requested they be repeated often, frequently reauditorized them, and had difficulty assimulating more than two actions within a single direction.

IMPRESSION:

Adam's achievement as displayed in this interview is consistent with school reports.

Impression, cont'd.

He seems to be performing in mathematics at the third or early fourth grade level. It should be noted, however, the concepts and skills he knows are quite well established.

Adam's overall "math sense" within numerical or geometric contexts was good. His ability to formulate and pursue multi-step strategies was not clear since his language seemed to interfere with tasks requiring applications of logical thinking.

There were no serious memory problems noted in recall of facts, recall of arithmetic procedures or visual memory of simple designs. However, there seemed to be recall deficits in memory of directions, visual memory of complex designs, and memory of mathematical data.

The most significant concern is the degree to which the nature of his language problems might interfere with his learning mathematics. The late third grade and future mathematical procedures require the integration of many steps (e.g., long multiplication, long division, fraction operations). The challenge will be for him to interpret these rather complex instructions into simple linguistic descriptions.

Stylistically, Adam prefers to deal with perceptual cues and appreciation of overall relationships and thrusts. The previously noted language deficits not only indicate impaired left hemispheric functioning, but also suggest continued learning problems as a secondary effect. Since school instructional programs and school functioning in general are so reliant on verbal interaction, language impairment has global impact on the full range of academic topics, including mathematics. However, the study of mathematics is often a source of fulfillment for language impaired children, since mathematics can so easily be interpreted through models which are primarily visual or perceptual in nature.

RECOMMENDATIONS:

The degree of stability and understanding of the concepts and procedures which Adam now demonstrates suggest that past instructional approaches have been effective and are appropriate.

Children with recall deficits and language problems often benefit from instructional approaches that emphasize the development of mathematical concepts and procedures through the manipulation of concrete materials. Careful transitions can then be made from the conceptual aspects of the activity at the concrete level to a more precise recording of the activity with abstract, numerical symbols. The key for Adam is to rely more on visual techniques than language techniques. Not only do such approaches emphasize the application of concepts, but they offer a model, as an alternative

Recommendations, cont'd.

to abstract memory for the specific components of a concept or procedure.

Adam's language deficits also suggest the need for instructional strategies which are segmented into meaningful, but smaller components which can then carefully sequenced.

Increased instructional time might be beneficial to Adam, since multiple opportunities to present the instructions would be available. Tutorial help by an individual sensitive to the needs of a language impaired child might be considered. However, the tutorial help should be provided within the current school setting.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Alex
D.O.B.: 6/19/67
C.A.: 11-11
Date of Evaluation: 5/24/79
Grade: 6

Alex was interviewed to assess his achievement, ability, and learning style in mathematics. In recent neurological and psychological testing reports, the findings indicated a generalized unevenness, very high performance in some areas coupled with relatively low performance in others. Arithmetic problems were characterized as difficulty in breaking down multi-step procedures and in carefully copying materials.

The most notable findings in the neurological evaluation were the marked difficulties with production of responses and output skills. Attentional and sequencing deficits were noted in a boy who simultaneously was described as "very bright."

DISCUSSION:

Oral Counting: Appropriate Responses

Alex performed all counting forward activities well. When asked to count backwards, Alex omitted all the multiples of ten, suggesting that he was responding to the "9, 8, 7, 6 ..." pattern without regard to the crossing points to next decades.

In the skip counting exercises, Alex could easily count by tens and by fives. When asked to count by twos, Alex started correctly. However, in trying to respond quickly, he reverted to counting by ones. He self-corrected, but then later reverted back to counting by ones. When asked to count by tens starting at seven, Alex appreciated the pattern of increasing numerals in the tens place and rapidly produced the sequence.

Alex clearly appreciated how each of the sequences was generated, but he was somewhat imprecise in producing the exact elements of particular sequences.

Basic Quantitative Concepts: Appropriate Responses

All basic quantitative concepts seemed well established. Alex was equally comfortable with both discrete (set) and continuous (rod) models for number.

Numerical Symbolism: Appropriate Responses

Alex could read and write numerals into the tens of thousands. In the dictated arithmetic problems, Alex correctly remembered the numerical components and organized the components correctly. However, he omitted all operations signs.

Alex could read and write all numerals for fraction numerals and decimal fraction numerals.

Arithmetic of Whole Numbers: Some Deficits

Alex demonstrated automatic recall of all addition, subtraction and multiplication facts.

Alex could correctly implement the addition and subtraction procedures both with and without regrouping. Alex could also correctly implement the multiplication and division algorithms. The answers were produced accurately and within acceptable time limits.

Alex did have difficulty performing a series of operations in his head. The multitude of stimuli seem to confound Alex.

Arithmetic of Fractions and Decimals: Appropriate Responses

Alex could correctly describe all operations involving fractions and decimals. He implemented these procedures well.

Informal Geometry: Area of Relative Strength

Alex was generally familiar with basic geometric shapes. However, he was imprecise in associating the correct name with the appropriate shape.

In the activities involving visual memory of geometric configurations, Alex was quick and accurate in reproducing the designs. Alex seemed to appreciate the overall design as well as the internal components.

In activities presented on the geoboard, Alex correctly copies figures in their appropriate orientations. When asked to find areas of regions, Alex used estimation techniques and perceptual judgments in making responses. Alex seemed reluctant to employ careful, deductive techniques which would yield more exact responses.

The facility and speed with which Alex dealt with geometric situations would suggest particular skills with perceptual stimuli and general appreciation of situations. The few errors that were observed were primarily those of imprecision.

Intuitive Problem Solving: Area of Relative Strength

Alex demonstrated excellent intuitions and strategies in informal problem solving situations. Alex did best in discerning governing patterns in analogies and sequences of shapes. Alex was less efficient in applying arithmetic skills to real-life situations. He correctly selected the appropriate arithmetic operations, but often incorrectly computed the answer.

When problems required careful analysis of internal details, Alex again was less efficient.

When iterative procedures were necessary, Alex needed some gentle external support in order to follow through to correct solutions.

Behavioral Aspects:

Alex was a delightful and charming boy. He responded very well in this one-to-one setting. Throughout the entire interview, Alex enjoyed engaging in relaxed conversations with the examiner. During these conversations Alex was, in fact, eliciting subtle structure from the examiner. This conversational approach also resulted in relaxing time constraints, allowing Alex additional time for deliberation and opportunities to deal with inherent details.

In some situations where there was a multitude of cues presented, Alex seemed a bit overwhelmed, and some tendencies towards perseveration were noted. Throughout many activities Alex demonstrated impatience with an overload of stimuli or the need for precise answers or exacting procedures. With external support, however, he was able to discipline himself to complete the tasks successfully.

Alex seemed to put a high premium on speedy responses, even to the extent of somewhat slurred speech. At times thoughts seemed to tumble out in an unorganized fashion. This need for speed is counter to Alex's natural need for deliberateness. Thus a real dilemma between perceived needs and natural instincts seems to exist.

IMPRESSION:

Alex's achievement in arithmetic is well within acceptable limits. Several concepts and procedures he has acquired are often not established until later grades. At times Alex's responses were not precise. However, there was no doubt that concepts and procedures are well understood.

Alex's performance in geometric activities seems to indicate an area of strength. His intuitions and skills are good. He utilizes perceptual cues and physical transformations of figures well in both two dimensions and three dimensions. Alex has not, however, acquired adequate geometric vocabulary. He seemed forgetful of detailed descriptions or precise terms.

Stylistically, Alex seemed comfortable with both verbal and perceptual stimuli. However, Alex's responses within perceptual contexts were often quite elegant. In formulating strategies, Alex seemed oriented primarily to overall thrusts and appreciation of the global situations. Unfortunately with this general focus Alex sometimes lost some of the salient details. Furthermore, inattention to detail manifested itself in imprecise, apparently "careless" errors in arithmetic.

When Alex was stressed by a multitude of stimuli or procedures with exacting subroutines, the correctness of his responses deteriorated. With external support, he proceeded well. It should be noted that Alex's major difficulties were in the organization of responses. Effective thinking patterns and understanding of concepts seemed well established. Unfortunately, it often did not emerge unless external structure and cuing were provided.

Alex's situation is further compounded by a tendency for deficits in actually producing the solutions he has indeed generated. At times he needed encouragement and external monitoring to elicit his strategies. Since he himself seems somewhat impatient with detailed solutions and inherently in need of speedy delivery of responses, he often gives somewhat impulsive answers and does not self-monitor their quality. With a minimum amount of external structure, he can bring his responses into sharper focus. Unfortunately, he does not spontaneously generate this behavior.

Alex now seems to be manifesting a lack of confidence in his mathematical capability, a posture unsupported by his actual level of achievement. Nonetheless, his "wrong answers" seem to suggest to him defective thinking strategies. This negative self-image, though apparently not now adversely affecting Alex's achievement, could affect future effort and achievement.

Alex demonstrated few memory deficits within mathematical contexts. His recall of arithmetic facts, recall of multi-step procedures, recall of directions, recall of mathematical components of situations and visual memory were intact. Alex's recall of arithmetic facts deteriorated when the facts were used in the formal algorithms. It seemed when Alex's attention to precise facts was diverted to the details of the required procedures, his accuracy suffered.

Alex's overall "math sense" is a source of real encouragement. His intuitions and selections of appropriate strategies suggest a boy whose overall ability is strong as well as a boy who appreciates mathematical concepts and procedures.

Interestingly, Alex's profile suggests the curious nature of mathematics itself. Mathematics (achievement), sensitive to organizational and production deficits, is also sensitive to good thinking strategies (ability).

RECOMMENDATIONS:

Alex has learned mathematical concepts and procedures well, despite a tendency toward imprecise answers. It would seem that the additional support and monitoring provided in his tutorial sessions have offered him the vital characteristics necessary for his good thinking to be developed. These characteristics are external support, monitoring, careful pacing and capitalization on his excellent general understanding.

Future academic programs should emphasize this supportive, personalized approach. Alex's excellent intuitions and strategies will need to be refined with some external intervention for productive responses.

Children with learning styles similar to Alex's often benefit from instructional approaches that emphasize the development of mathematical concepts and procedures through the manipulation of concrete materials. Careful transitions can then be made from conceptual aspects of the activity at the concrete level to a more precise recording of the activity with abstract, numerical symbols. Not only do such approaches emphasize the application of concepts, but they offer models which may serve as alternatives to abstract memory for the specific components of a concept or procedure.

The choice of mathematical model is especially important in view of Alex's preferred learning style. A continuous model, such as rods, would seem most appropriate. This model appeals strongly through perceptual cues for concepts and procedures under study.

However, when such a model is presented, strong external structure and monitoring are required, since children whose styles seem to respond best to those models cannot spontaneously appreciate details or organize their strategies.

Since algebraic and numerical work is more significantly concerned with detail and precise deductive processing, young adults with profiles similar to Alex's often find themselves handicapped in these topics. Furthermore, geometric topics, as presented in many high school curriculums, are themselves highly deductive, requiring precision in their details, despite the perceptual stimuli. Thus, this course tends to be adversely affected by the particular learning style Alex and others like him display. All this suggests that Alex's progress will need to be continually monitored to insure that instructional approaches are appealing to his particular learning style.

Standardized tests would tend to minimize Alex's true mathematical achievements and ability. Alex's learning style, lack of confidence, and general impatience with details would cloud his valid responses and his appreciation of the general flavor of the correct responses. Furthermore, such tests direct more attention to arithmetic solutions and geometric vocabulary, the very areas in which Alex demonstrates most of his weaknesses. Thus, scores on standardized tests would be generally depressed. An indication of his strengths in considering whole situations, perceptual cues and geometric applications would not be evident.

Alex's excellent problem solving ability should be encouraged and exercised through informal games and puzzles. With successes in these activities, Alex's confidence might also improve.

To re-emphasize, Alex's progress should be carefully monitored to insure his potential is achieved and reflected in the production of responses.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Julia
D.O.B.: 4/29/66
C.A.: 13-2
Date of Evaluation: 6/26/79
Grade: 8

Julia was interviewed to assess her achievement, ability, and learning style in mathematics. School reports note general academic difficulties with below grade level performance in both reading and mathematics. Last year she was in the "C" mathematics group (of groups A,B,C). Julia was described as much better than other students in the group in some aspects, especially geometry, while at the same time having more difficulty than many of them, particularly in computational skills. It was reported that Julia enjoys and does especially well in mathematical activities that involve the manipulation of concrete materials. A particularly successful experience with geoboards was described by her teacher.

DISCUSSION:

Oral Counting: Appropriate Responses

Julia responded well to al' :ounting activities. When asked to count by 10's starting at 7, Julia responded particularly rapidly, suggesting that she recognized and utilized the governing pattern.

It is also noteworthy that when Julia was counting backwards, Julia was particularly hesitant when crossing the decades (50, 40, 30, 20), once again suggesting that she was responding to the governing pattern. She is able to monitor her responses in this activity so that she is aware of the details of the individual elements of the sequence.

Basic Quantitative Concepts: Appropriate Responses

All basic quantitative concepts seem well established. Julia can interpret numbers

with both a discrete (set) and continuous (rod) models. Qualitatively, her responses seemed more elegant when using the continuous model.

Numerical Symbolism: Appropriate Responses

Julia could read and write numerals into the tens of thousands. She could read and write fraction numerals. Julia could write decimals but had difficulty reading them correctly.

In the dictated arithmetic problems Julia frequently used reauditorization to help her recall the elements.

Arithmetic of Whole Numbers: Significant Deficits

Julia demonstrated little automatic recall of any arithmetic facts. However, she has developed nice compensatory strategies to generate unknown facts. In addition, her technique was to build tens. In subtraction she seemed to be using a missing addend technique. Nonetheless, Julia's responses to arithmetic facts were accurate albeit deliberately delivered. The speed with which she produced multiplication facts, again accurately, was especially slow.

Julia could apply the addition algorithm well both with and without regrouping. When subtraction of two-digit numerals was presented, Julia never once used the traditional algorithm. She used a 'building tens' technique which involved missing addends. Unfortunately, Julia did not implement her technique well and was often imprecise in her responses. Nevertheless, she always sensed a totally incorrect solution.

Julia seemed to do better when writing down her calculations rather than attempting them mentally. The actual recording of steps seemed to provide some organization for the procedures.

Julia could apply the multiplication algorithm with one-digit and two-digit multipliers.

Julia was generally familiar with the division algorithm, but her solutions were imprecise.

When implementing arithmetic procedures, Julia appeared to be talking herself through recipes. This technique seemed to require particular effort on her part.

Arithmetic of Fractions and Decimal Fractions: Significant Deficits

In the operations for fractions and decimals, Julia could verbally described the particular rule for most procedures. However, she frequently implemented the procedures incorrectly. The details and organization implicit in the pursuing of the actual steps seemed especially difficult for her.

In general, all basic arithmetic concepts and procedures seemed generally established. Julia's responses were always "in the ball park." However, precise solutions often eluded her. Julia seemed particularly troubled by dealing with the specific details and in organizing them for a successful conclusion.

Informal Geometry: Area of Strength

In all activities involving visual memory of geometric configurations, Julia responded quickly and accurately. She appreciated the overall configuration as well as the internal components.

On the activities with the geoboard (a medium which was described as one of Julia's favorite activities), Julia again responded well.

When copying geometric figures in either medium, Julia used reauditorization techniques when complex designs were presented. These techniques seemed helpful to Julia.

In activities involving the copying of three-dimensional configurations of cubes from pictorial cues, Julia appreciated each dimension individually but had real difficulty in integrating them into a meaningful whole. She recorded the dimensions in her own drawing (which was nicely produced). However, once the labeled figure was before her she was seduced to a perimeter interpretation. With a great deal of external monitoring and help, along with the opportunity to actually build the configuration with cubes, Julia came to appreciate the three dimensions as they related to each other. From there on, she was able to interpret all other similar activities.

This performance suggested Julia's real need to interpret situations concretely in order to assimilate multiple cues. Furthermore, it demonstrated how Julia, having initially missed the relationship between internal components, has difficulty refining her approach in abstract terms.

It should be noted that Julia is aware of her errors and difficulties. However, without concrete interpretation, she could not effectively modify her approaches.

In the geometric problem solving activities Julia quickly aporeciated the strategies necessary to approach the problems. She then followed through effectively.

Intuitive Problem Solving: Area of Strength

Julia could apply arithmetic techniques to solving real-life situations. In more complex situations, Julia needed some external structuring to refine her initial approaches to yield successful solutions.

Julia very quickly discerned patterns governing analogies and sequences. Her intuitions and strategies were excellent.

Behavioral Aspects:

Julia was friendly, poised and thoroughly charming throughout the entire interview. Her positive attitude towards mathematical activities, despite her own awareness of calculation difficulties, is a tribute to Julia herself and her recent mathematics learning experiences.

IMPRESSION:

Julia's achievement in arithmetic is seriously below grade level in terms of correct responses. However, all underlying concepts and procedures are understood. The difficulties seem to arise in implementing these procedures precisely and quickly.

Julia demonstrates selected memory deficits within mathematical contexts: recall of arithmetic facts and recall of complex verbal instructions. Her recall of procedures, visual memory and mathematical components of situations were encouraging. At times Julia uses reauditorization to cue her. This strategy seems taught, rather than instinctive but is, nonetheless, effective.

Stylistically, Julia seems to be able to appreciate quickly overall thrusts and the totality of situations. She is particularly facile with highly perceptual cues. However, in more complex situations where attention to salient details is a key to successful solutions, Julia had difficulty organizing, structuring and integrating those details. Verbal external intervention was less helpful than concrete interpretations. Where Julia could build prototypes, Julia established firm understanding and successful strategies.

Julia's excellent intuitions and strategies become apparent within geometric and logical contexts where calculation details do not inhibit her. Julia's "math sense" seems to emerge in these situations and seems to be a real area of strength.

Julia's learning style puts her at real "risk" for failure in future traditional mathematics courses, where the emphasis is often on details and precision. Julia's excellent "math sense" places her in a position of promise for dealing with mathematics in life's environment.

Standardized tests will tend to minimize Julia's true level of competence in mathematics. Only in less formal testing will her strengths emerge.

RECOMMENDATIONS:

Julia's particular learning style warrants a highly personalized mathematics program. The emphasis in the instructional approaches should be supportive, helping Julia bring her talents to bear on given situations while offering her external monitoring and structuring.

In arithmetic the emphasis should be on estimation techniques and understanding how arithmetic procedures can be applied to real-life situations. The burden of pr :ise calculation can be reduced through the introduction of a simple calculator. However, before a calculator can be effectively used, estimation techniques and understanding of the operations themselves must be established.

In future topics, such as algebra, Julia will need special consideration in terms of time allowances, additional opportunities for instruction and interpretation in concrete terms. Even then, these topics will be treacherous for Julia. However, her positive attitude and persistence are powerful factors.

In the meantime, Julia's excellent geometric and logical intuitions and strategies should be exercised and encouraged.

Well down the road, a special tutorial in geometric topics might be substituted for the more rigorous and traditional Euclidean geometry course. The tutorial might focus on the geometric characteristics and relationships inherent in our environment, e.g. symmetry, transformations, topology.

MATHEMATICAL DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Michael
D.O.B.: 4/20/65
C.A.: 14-4
Date of Evaluation: 8/31/79
Grade: 9

Michael was interviewed to assess his achievement, ability, and learning style in mathematics. Michael is reported to have persistent primary encopresis, mild learning problems, and school failure.

In early evaluations Michael was characterized as having poor attentional skills, problems with the output of language, problems with auditory and visual sequencing, and poor achievement in mathematics. Special tutoring was suggested for mathematics.

Currently, Michael is under care for his problems with encopresis. Michael reports his vocational goal is to be a policeman, as his father is.

DISCUSSION:

Oral Counting: Appropriate Responses

Michael responded well to all counting activities. When asked to count by tens, starting at seven, Michael initially had difficulty. He at first responded "7, 14, 21" but realized this was not correct. It wasn't until he wrote down each element and mentally added ten, that he recognized the governing pattern. In order to achieve success, strong external structuring by the examiner as well as the written form was necessary.

Basic Quantitative Concept: Appropriate Responses

Most basic quantitative concepts seemed well established. However, Michael demonstrated persistent difficulties with precision and with verbal directions. For example, when asked for a number less than "65," he responded "69", then self-corrected to "39".

Michael could interpret place value with both discrete (chip) and continuous (rod) models for number. However, he was initially hesitant and deliberate.

Michael could read and write whole numerals into the tens of thousands. He could read and write fraction numerals adequately. Michael had real difficulty reading and writing decimal numerals.

In the dictated arithmetic problems, Michael demonstrated appropriate recall and organizational skills.

Arithmetic of Whole Numbers: Significant Deficits

Michael demonstrated adequate recall of addition and subtraction facts. However, when less familiar facts were presented, Michael was quite hesitant, e.g."16-8". Eventually, he reached a correct solution. His deliberation suggested he was relying on compensatory strategies, rather than automatic recall. Michael's recall of multiplication facts was quite unstable. When he did not know a solution, he simply gave up. He demonstrated no tendency toward nor interest in applying compensatory strategies. Michael's control of division facts was similarly weak.

Michael could apply the addition and subtraction algorithms both with and without regrouping. Michael could apply the multiplication algorithm with a one-digit, but not a two-digit multiplier. Michael could not apply the division algorithm at all. Michael demonstrates few skills with estimation. He seems not to appreciate or evaluate the appropriateness of his solutions.

All in all, Michael's performance in arithmetic was like that of a late third grader.

Arithmetic of Fractions and Decimals: Significant Deficits

Michael demonstrated good use of fraction symbols, but could not apply any operations (+, -, x, ÷) procedures to fractions.

When given problems with decimal numerals, Michael rewrote each numeral as a fraction and then attempted to proceed.

Generally, no competence with fractions and decimals were noted.

Informal Geometry: Area of Relative Strength

Michael was familiar with all basic shapes. In the activities involving visual memory of geometric configurations, Michael was generally accurate. On more

complex designs he required additional cuing, but then completed the designs appropriately. Michael's initial errors were in the organization of the components.

In the activities on the geoboard, Michael again demonstrated initial organizational problems. However, with repeated opportunities he achieved success. Interestingly, Michael often actually placed himself at 90° rotation to the stimulus presented.

In the informal problem solving within geometric contexts, Michael formulated highly segmented strategies. Often his perceptual judgments were inaccurate. In more complex tasks, Michael had a tendency to just give up.

Intuitive Problem Solving: Basic Level of Performance

All responses Michael gave in these activities were marked by imprecision. It should be noted these were the last tasks presented in a one-hour session. Michael's imprecision seemed to result not only from his particular learning style, but also from his inability to sustain attention and effort for long periods of time.

In the activities involving applying arithmetic skills to real-life situations, Michael needed to draw pictorial cues to help himself reach solutions.

In the activities which require that patterns governing analogies, sequences or sets of elements be discerned, Michael responded adequately when the cues were simple. As more stimuli were to be integrated, Michael had increasing difficulty. He often was attracted to one characteristic at the expense of other salient qualities of the situation.

Behavioral Aspects:

Michael was cooperative and hard working during the entire interview. He was quite serious and conversational in a "pseudo-adult" manner. The quality of his social interaction, although welcome, was not typical of teenage boys.

Michael had persistent difficulty with verbal directions, frequently asking for repetition or clarification. External structuring was also frequently required in order to help Michael organize his strategies and responses.

IMPRESSION:

Michael's achievement in arithmetic is noteworthy for its serious deficiencies.

Specific problems with multiplication and division concepts and procedures are of real concern. Furthermore, deficits with decimals, with the coming of our conversion to metric measurement are particularly noteworthy.

Michael demonstrated selected memory deficits within mathematical contexts: recall of arithmetic facts, recall of multi-step procedures and recall of directions. His recall of mathematical components of situation as well as his visual memory seemed intact.

Stylistically, Michael seemed to approach situations in a part-by-part fashion. However, he often had difficulty organizing and implementing those components. Furthermore, complexity seemed to stress Michael's effectiveness.

Michael did his best work in concrete or pictorial modalities. Highly verbal or highly abstract stimuli posed particular problems for him.

Some interesting behavioral characteristics should be noted. Michael had persistent difficulty with verbal directions or instruction. When situations became too complex, Michael seemed overwhelmed and gave up. His impersistence was often marked by overt confidence and impatience with "unimportant detail."

RECOMMENDATIONS:

Michael's achievement in mathematics was significantly deficient with specific weaknesses in estimation, multiplication, division, fraction and division skills. Michael's present performance could not warrant a description of Michael as mathematically literate.

The first task of any mathematics program would be to enhance those skills with particular emphasis on understanding the concepts underlying each operation, the appropriate applications of each operation and the skill to estimate results.

Once these foundations have been established, a simple calculator might be introduced. This device would relieve Michael of the need for precision and recall of multi-step procedures, while increasing the speed and accuracy of his responses.

Such instructional programs should be highly personalized and on a small group basis in order to accommodate Michael's particular learning style and behavioral postures.

At Michael's age, further formal study in mathematics should be based on educational and vocational goals. Michael's present learning style and favored modalities would make studies like formal algebra and Euclidean geometry quite stressful and difficult for him.

However, Michael's progress and growth should be carefully monitored. A full reevaluation in one year would be appropriate. Then Michael's growth could be evaluated and his goals further refined. The advisability of formal mathematics study and the conditions under which it would be successful could then be discussed.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: John
D.O.B.: 12/29/62
C.A.: 16-2
Date of Evaluation: 2/16/79
Grade: 10

John was interviewed to assess his achievement, ability, and learning style in mathematics. John is currently taking Algebra II. Though his grades have been adequate, his parents were interested in having him tested because of his tremendous difficulty with certain topics such as word problems and application of formulae.

Previous Testing: None

DISCUSSION:

Oral Counting: Appropriate Responses

Basic Quantitative Concepts: Appropriate Responses

Informal Geometry: Appropriate Responses

John's visual memory appeared to be intact as he was able to reconstruct geometric designs. He was able to name even less familiar shapes such as trapezoid and parallelogram.

John appears to have limited strategies and intuitions when approaching geometric problems. In solving informal area problems on a geoboard, John needed some prompting as to how to approach the problem, but then was able to proceed on his own.

John seemed to approach the configurations through their individual components. He generally had little difficulty remembering or organizing components. However, on

Informal Geometry, cont'd.

complex configurations, John had some difficulty assimilating the halves of the designs.

Arithmetic of Whole Numbers: Strong Performance

John had automatic recall of his facts in the four basic operations. He also knew all the algorithms and performed even long division faultlessly.

Arithmetic of Fractions and Decimals: Strong Performance

John read decmial notation correctly. He was also able to perform all operations with both fractions and decimals without reminders and without errors.

Algebra: Some Deficits Apparent

John demonstrated good skills with integers. He could interpret and operate with them accurately. He interpreted the $\sqrt{}$ symbol accurately but needed help to simplify more complex radical expressions. He confused the fractional exponent notation with the meaning of negative exponents. John could solve linear, quadratic and fractional equations and also pairs of equations with two variables. He was familiar with and comprehended well algebraic terminology such as "factor," "expand," "simplify" and "solve."

John did experience great difficulty in solving word problems using algebraic techniques. He was not able to determine the unknowns or set up an equation.

Intuitive Problem Solving: Appropriate Responses

John can appropriately apply arithmetic procedures to real-life situations. He can discern patterns between objects in a sequence and he can determine the governing relationship between pairs of numbers and sets of objects.

BEHAVIORAL ASPECTS:

John's behavior during the entire interview was friendly and cooperative. John appeared to be a mature, serious, conscientious student. What he considers to be a mediocre past experience with mathematics has given him a poor self-image with regard to his mathematical competence. This attitude contributes to the very lack of success that generates it.

IMPRESSION:

John's achievement in arithmetic, fractions and decimals, and algebra is securely established at grade level. John needs to be made aware of the high level of his accomplishments and encouraged to think well of his achievement. He demonstrates not only rote learning of basic concepts but also real understanding of these concepts.

Stylistically, John seemed to approach situations through their components which he then organized in a step-by-step fashion. Analytically, John also seemed to prefer a step-by-step, deductive strategy. The stimuli, which pervade these strategies and to which John responded most favorably, were verbal in nature. Thus, John's success with algebraic topics is not surprising, since algebraic procedures lend themselves so easily to verbal interpretations. The key to success seems to be the ability to describe each procedure and the circumstances to which it applies verbally. Then, a careful step-by-step implementation of the description leads to correct responses. Generally, teaching techniques in algebra classes also nurture these strategies.

Students whose learning styles are similar to John's seem to encounter difficulty when strategies are not easily verbalized or when overall appreciation of a situation precedes selection of a particular strategy. The decision as to which strategy is appropriate often calls upon a Gestalt "overall" feeling for a situation. These approaches, often inherent in informal geometry situations, did not seem easily accessible to John.

John's apparent difficulty with geometric problem solving seems ameliorated by external, verbal interpretation provided by the examiner. Furthermore, John's problems with real-life applications, even somewhat simple ones, reflect John's dilemma in choosing an approach from many available ones. Both these behaviors would confirm John's preference for deductive, step-by-step strategies, dominated by verbal cues (a profile often associated with left hemisphere functioning). When more overall appreciation of a situation or multiple strategies applied in parallel fashion is required, John is less facile. He is, however, comfortable with perceptual cues. (Right hemisphere functioning is often characterized by perceptual cues and parallel processing.)

RECOMMENDATIONS:

John's current level of achievement and understanding are sources of great encouragement. They reflect a bright boy who has learned well.

John's learning style is probably established by nature. In regard to hemispheric interpretations individuals do not necessarily have balanced performance with respect to intellectual functioning. Therefore, John's lack of facility with inductive processes can be viewed simply as a part of his makeup. He will naturally revert to

Recommendations, cont'd.

the style which is most comfortable for him, i.e., deductive approaches with a high level of verbal interpretation. Teachers should understand John's particular profile and should help John develop verbal interpretations or step-by-step strategies to approach broader situations. Once these more detailed strategies are established, John's apparent lack of application or direction will lessen. Furthermore, John will become confident that he can initiate an action. Often students like John become discouraged when they sit facing a problem with a blank piece of paper, and no first step emerges. A set of procedures or structural cues to analyze a situation as well as to proceed to a solution is a great comfort and enhances self-image. Sometimes, John's need for step-by-step processing will result in more deliberate solution patterns, but he will be able to reach successful conclusion.

The outlook for John's achievement in geometry, a topic which usually emphasizes skills associated with right hemispheric learning styles, is nonetheless good. John demonstrates no problems with perceptual cues. Furthermore, formal geometry is very deductive in its requirements, in marked contrast to its more inductive nature as it exists in our environment. At times, some additional verbal cues may be provided by John to interpret geometric procedures. Some strategies may need to be analyzed into smaller components in order for John to assimilate them.

For more long-term predictions, John will, no doubt, naturally select educational and vocational pursuits for which his learning style seems comfortable. With John's overall ability, a successful future might well be anticipated.

MATHEMATICS DIAGNOSTIC/PRESCRIPTIVE INVENTORY

SAMPLE CASE STUDY

Name: Paul
D.O.B.: 4/30/59
C.A.: 18
Date of Evaluation: 4/6/78
Grade: 12

Paul was interviewed to assess his achievement, ability, and learning style in mathematics. He was referred for this evaluation in response to a history of academic problems in mathematics achievement.

Currently, Paul is a high school student with plans to go on to college in the fall. His stated college choices were University of Maine, University of Vermont, and New England College. His vocational interests at this time are in the areas of real estate, construction or business administration. At 18, Paul is presently enrolled in two mathematics courses: Geometry and Algebra II (on a tutorial basis).

DISCUSSION:

Numerical Symbolism: Some Deficits

Paul's responses contained vestiges of earlier difficulties with numerical symbolism. It appeared that he might have had particular difficulties with reversals. As he has matured, he has developed compensatory strategies to overcome these tendencies. His current symbolic responses are, however, marked by poor organization. He also demonstrates some difficulty with place value when applied to very large numbers and decimals.

Arithmetic of Whole Numbers: Some Deficits

Paul's overall performance in arithmetic is adequate and is developed to a level appropriate for utilization in life's situations. However, Paul demonstrates a less than secure recall of arithmetic facts and a tendency toward careless errors in arithmetic procedures. Furthermore, it seems necessary for Paul to subvocalize each step of the multi-step procedures in order to cue himself to the precise procedures.

Arithmetic of Fractions and Decimals: Significant Deficits

Paul demonstrates less than adequate skills with fractions. He has acquired rudimentary skills; however, they are not secure.

Paul's competency with decimals is very poor. He is unable to write decimals correctly, although he can read them adequately. Paul cannot perform the arithmetic operations on decimals. These deficits are of particular concern since decimals are destined to become a more integral part of our society, especially as it converts to the metric system.

Algebra: Significant Deficits

Paul demonstrates acceptable skills with integers. He can interpret and operate with them accurately. He cannot deal with radical expressions at all, misinterpreting the symbol and the fractional exponent notation.

Paul can evaluate simple, first degree algebraic expressions, but not second degree expressions.

He is unfamiliar with algebraic terms: "factors," "binomial," "solve." His skills in the areas of factoring expressions and solution of equations are non-existent. He also cannot simplify many algebraic expressions.

Overall, Paul's competency in algebra is severely deficient.

Informal Geometry: Appropriate Responses

Paul seemed unfamiliar with many common geometric terms. However, the manipulation of geometric figures was quite well developed and may even be an area of strength. His visual memory skills, figure/ground discrimination skills, and perceptual abilities seemed especially well developed. His strategies and intuitions within geometric problem solving situations were well executed.

In general, Paul seemed to prefer dealing with whole configurations rather than internal details. His analytic procedures seemed to be inductive in nature, dealing with the generality or total situation initially. He did demonstrate some lack of precision in integrating later detail work.

Behavioral Aspect:

Paul's behavior during the entire interview was friendly and cooperative. However, he displayed a very negative self-image with regard to his mathematical competence. He frequently said, "I don't know," or, "I could never do this." At times, he was reluctant to start an exercise which he later could do. He was impersistent in his work. He often gave up, thinking he couldn't successfully complete a task or thinking he was on the wrong track.

It is not unusual to find these characteristics and attitudes in young adults who have experienced a history of non-success in their mathematical courses. Unfortunately, these attitudes themselves contribute to the very lack of success that generates them.

IMPRESSION:

Paul's achievement in arithmetic is generally acceptable. He understands the underlying concepts and can apply arithmetic procedures. His deficits in organizational skills, in recall of facts, and in detailed computation can be easily compensated with the use of a simple hand calculator.

Paul's achievement in fraction work, decimal concepts, and decimal procedures is seriously deficient.

Paul's achievement in algebra is significantly deficient. He has attained few concepts or skills.

Paul's performance in geometric activities seems to indicate an area of strength. His intuitions and skills are good. He utilizes perceptual cues and physical transformations of figures well, in both two dimensions and three dimensions. He has not, however, acquired adequate geometric vocabulary. He seemed forgetful of detailed descriptions or precise terms.

Paul's overall "math sense" seemed stronger within geometric situations than algebraic ones. His intuitions and strategies seem to be especially well developed within sensory/preceptual modalities. His strategies and logical procedures seem to be inductive in nature. Paul's preferential style seems to be in dealing with wholes or total situations. From this perspective, he proceeds to the internal details or implications. However, imprecision with details seemed to be characteristic of all of Paul's thinking despite the area of mathematics. Since algebraic and numerical work are more significantly concerned with detail and precise deductive processing, young adults with profiles similar to Paul's often find themselves handicapped in these topics. Furthermore, geometric topics, as presented in most high school curriculums, are themselves highly deductive, requiring precision in their

Impression, cont'd.

details, despite the perceptual stimuli. Thus, this course tends to be adversely affected by the particular learning style Paul and others like him display.

Much current thinking and research lends itself to an interpretation of intellectual functioning that attributes particular skills and styles to each of the two brain hemispheres. Such descriptions note that, whereas the "localization" of physical skills seems to be symmetric with respect to brain hemisphere, intellectual skills seem to be asymmetric. That is, each hemisphere does not control corresponding abilities. Some activities seem controlled primarily by one hemisphere. It also does not seem that each hemisphere processes information in the same way. Thus, an intellectual function of a particular kind is often thought of as being mediated primarily by only one hemisphere. However, it must be noted that for the effective utilization of any concept or procedure, the activity of both hemispheres must be well integrated.

In considering mathematical ability within this framework, it must be noted that mathematics is comprised of many individual areas. This may suggest that mathematical topics might be located in different regions of the brain and, in fact, in different hemispheres.

Empirical data suggest the following characteristics of the right hemisphere of the brain with respect to mathematical achievement and ability. Tasks that seem to be mediated by the right hemisphere include: geometry, mapping and various other topics in which the stimuli are primarily sensory or perceptual. Furthermore, the right hemisphere seems to prefer to operate in a "gestalt" fashion. It deals with the totality of a situation. It attends to the overall configuration rather than individual details.

Paul's performance in this interview, when considered as a whole, seems to indicate a preferentiality toward right hemisphere skills and styles of processing. Fortunately, Paul's vocational interests are in areas that would favor this preferentiality. Some would argue that schools in their mathematical goals are somewhat discriminatory against this style. A preference for activities mediated by the right hemisphere would indeed be explanation for Paul's past failures in arithmetic and algebraic topics.

RECOMMENDATIONS:

Paul would increase his accuracy and precision in arithmetic with the aid of a calculator. The preliminary concepts necessary to use a calculator seem well

Recommendations, cont'd.

established. Further practice with estimation would enhance calculator applications.

Paul can de-emphasize the topic of fractions at this time. Rudimentary foundations have been established. Any further development of skills would best occur within the context of the real applications Paul encounters. Unfortunately, Paul's deficits in decimals cannot be treated as easily. A specific remediation program is warranted to increase Paul's "literacy" in this topic. With the coming of the metric system, some familiarity and agility with decimals will be mandatory.

Paul's algebra skills also require strengthening in order to establish basal skills necessary for entry into the fields of vocational interest he has indicated. He needs to be more familiar with the language of variables, simplifying expressions and simple solution sets.

A continuation of formal geometric study seems to yield few beneficial aspects. Paul's particular learning style is not consistent with more traditional geometric goals. Although applied geometric topics and sensory/perceptual stimuli are areas of strength for him, they do not usually play important roles in an initial Euclidean geometry course.

Paul's choices of vocational possibilities seem to be consistent with his preferred learning style. Real life applications and appreciation of overall aspects of a problem are often important elements in construction and real estate work.

Paul's negative self-image in mathematics will only improve with experiences of success. Current instructional programs should be on a one-to-one basis to provide the support Paul needs and in order to tailor situations in such a way so that Paul can learn and succeed. His attitudes of impersistence, "giving up" and deliberation of response can only be improved by external monitoring and encouragement.

Standardized tests would tend to minimize Paul's true mathematical achievements and ability. Paul's learning style, lack of confidence, and general impatience with details would cloud his valid responses and his appreciation of the general flavor of the correct response. Furthermore, such tests direct more attention to arithmetic, algebraic, and geometric vocabulary, the very areas in which Paul demonstrates most of his weaknesses. Thus, scores on standardized tests would be generally depressed. An indication of his strengths in considering whole situations, perceptual cues, and geometric applications would not be evident.

SECTION II

METHODS USED IN THE STUDY

- SUBJECTS
- MEASURES AND PROCEDURES
- VARIABLES
- OVERVIEW OF RESEARCH DESIGN

SECTION II

METHODS USED IN THE STUDY

Subjects

The individuals included in this study come from the population of individuals referred to the Learning Disabilities Clinic, Department of Neurology, Children's Hospital Medical Center, Boston, Massachusetts. These individuals are accepted by the clinic if they are reported to have global learning disabilities. Individuals who are thought of as having more specific disabilities, such as "emotional disturbance," mental retardation, or seizures, are generally seen in another more specialized area of the hospital.

In the realm of research in medicine, studies have often been centered on unique populations who have known deficits. The style of research is to relate the observed deficits to known physical conditions. By implication, the physical conditions associated with the loss of effective function suggest that effective function is associated with the correspondingly "healthy" physical aspect.

In a similar fashion, in order to focus on mathematical learning, this project utilized individuals who have not learned well. Those areas of observed weakness in the learning style were then associated with the absence of specific skills or deficits on specific items. By implication, those specific skills or items were assigned to a specific learning profile.

The subjects included in the study were the population of 220 students who were seen at the clinic during the three-year period, 1977-1979, that Martha Bridge Denckla, M.D. was Director of the Clinical Team. Some specific characteristics of the subject population are given in the following charts.

Specific Characteristics of Subject Population

TOTAL NUMBER = 220

Boys :	162	73.6%
Girls :	58	26.4%

AGE	N	% of Population	Cumulative %
5	1	.5	
6	10	4.5	Ages 5-8 28.2 %
7	22	10.0	
8	29	13.2	
9	34	15.5	Ages 9-12 53.7 %
10	34	15.5	
11	26	11.8	
12	24	10.9	
13	12	5.5	Ages 13-17 18.2 %
14	12	5.5	
15	2	.9	
16	8	3.6	
17	6	2.7	

GRADE	N	% of Population	Cumulative %
K	5	2.3	
1	19	8.6	Grades K-2
2	23	10.5	21.4%
3	40	18.2	
4	32	14.5	Grades 3-6
5	36	16.4	59.1%
6	22	10.0	
7	12	5.5	Grades 7-8
8	12	5.5	11.0%
9	8	3.6	
10	8	3.6	Grades 9-12
11	3	1.4	8.6%
12	—		

Arithmetic Achievement as Reported by Teacher — (N = 220)

Delayed More Than One Year	65	=	29.5%
Delayed One Year	58	=	26.4%
Appropriate Level	48	=	21.8%
Strong for Age	7	=	3.2%
No Information	42	=	19.1%

Reported Previous Testing — (N = 220)

None Reported	159	=	72.3%
WRAT	18	=	8.2%
Key Math	12	=	5.5%
Stanford Achievement	9	=	4.1%
SRA	5	=	2.3%
Other	17	=	7.7%

Reported Results on Previous Testing — (N=220)

No Data	159	=	72.3%
Delayed More Than One Year	19	=	8.6%
Delayed One Year	17	=	7.7%
Appropriate Level	21	=	9.6%
Strong for Age	0	=	0.0%
Other	4	=	1.8%

Reported Ability (WISC - Previously Admin.)

	Verbal	Performance	Full Scale
62 - 69:	1 = 1%	5 = 3%	2 = 1%
70 - 79:	13 = 8%	5 = 3%	10 = 7%
80 - 89:	32 = 21%	27 = 18%	26 = 17%
90 - 99:	41 = 27%	40 = 26%	48 = 32%
100 - 109:	33 = 22%	44 = 29%	35 = 23%
110 - 119:	17 = 11%	22 = 14%	21 = 14%
120 - 129:	12 = 8%	6 = 3%	7 = 5%
130 - 139:	3 = 2%	5 = 3%	2 = 1%
	N=153 = 100%	N=153 = 100%	N=151 = 100%
No Data	67	67	69

Measures and Procedures

The individuals in this study were evaluated by a team of specialists in the medical, psychological, and educational fields. The pertinent measures and findings selected for this study included

Medical/Neurological: Findings of a neurologist based on the classical neurological examination of motor system and an extended neurological examination of cerebral/motor integration.

Psychological/Neuropsychological: Findings of a neuropsychologist based on such measures as selected subtests of the WISC-R, the Visual Motor Integration Test, the Rey Osterreich Complex Figure Design, and standard and non-standard sequences.

Mathematical Skills and Procedures: Findings from the Mathematics Diagnostic/Prescriptive Inventory.

No special data were collected for the sake of the study. The information gathered was taken directly from the hospital reports filed on each subject which included detailed written reports from each of the medical/educational team members (neurologist, neuropsychologist, psychiatrist, audiologist, speech and language specialist, reading specialist, and mathematics specialist), together with the case coordinator's summary of the findings.

Since no similar study had ever been carried out on these data, many steps had to be taken to develop the data base and variables to be studied and analyzed from so much possible information on each subject. The procedures which were devised and followed are listed in chronological order:

A. Development of the Data Base

1. Establishment of the list of names of the subjects seen at the Learning Disabilities Clinic, Children's Hospital Medical Center, within the designated time period

2. Coding of the names to three-digit numerals for the purpose of confidentiality and computerization

3. Assembly of the pertinent raw data available from the hospital records for each subject:

   Case summary
   Family history
   School report
   Neuropsychologist's report
   Neurologist's report
   Mathematics report
   Mathematics raw data recording sheets

4. Consultation with the hospital professionals regarding the raw data:

   Neuropsychologist
   Neurologists
   Educational Coordinator
   Administrative Assistant

5. Consolidation of the list of subjects to a total of 220 with complete data bases

B. Development of Mechanics for Recording the Data

1. The formation of independent files on each of the 220 subjects with Xeroxed copies of the pertinent raw data

2. Consultation with the Administrative Staff for the ongoing compilation of data:

   Head of the Learning Disabilities Clinic
   Statistical Consultants to the Project

C. Establishment of Categories for Case Histories

1. Review of raw data from

   School reports
   Results of achievement tests
   Hospital findings in the areas of math, reading, and emotional issues

2. Consultations with various public school staff to discuss school reporting

D. Establishment of Categories for Neuropsychology

1. Reading of the neuropsychological reports

2. Consultation with the neuropsychologist

3. Establishment of categories for the neuropsychological findings

4. Approval of the categories by the neuropsychologist

E. Establishment of the Categories for Neurology

1. Reading of the neurological reports

2. Consultation with the neurological staff

3. Establishment of categories for the neurological findings

4. Approval of the categories by the neurological staff

F. Establishment of Categories for Mathematics

1. Establishment of categories and preliminary coding scales for mathematics by age levels 05-08, 08-12, 12-Adult in the form of profile summary report forms

2. Administration of the Mathematics Diagnostic/Prescriptive Inventory and use of the Profile Summary Report forms on a trial basis with new subjects (apart from the study)

G. Establishment of a Coding Key for All of the Categories of Data (See Volume II, Section IX of this report)

Case Histories (General Data)
Neuropsychological Data
Neurological Data
Mathematics Profile Data

1. Establishment of Standard Rating Scales for Neuropsychology and Neurology

Standard Scales for Neuropsychology and Neurology

01 Data implied significant compromise

02 Data implied some compromise

03 Data implied appropriate

04 Data implied some strength

05 Data implied significant strength

09 No data

11 Reported significant compromise

12 Reported some compromise

13 Reported appropriate

14 Reported some strength

15 Reported significant strength

2. Consultations with Statistical Consultants, Neuropsychologist and Neurologist regarding Rating Scales for Neuropsychology and Neurology

3. Refinement and consolidation of the mathematics summaries and the establishment of Standard Rating Scales for the Mathematics Profile Data

Standard Scales for Math

01 Assumed no competence

02 Assumed significant deficits

03 Assumed some deficits

04 Assumed appropriate responses

05 Assumed better than expected

06 Assumed area of strength

09 No bases for assumption

11 Tested no competence

12 Tested significant deficits

13 Tested some deficits

14 Tested appropriate responses

15 Tested better than expected

16 Tested area of strength

H. Establishment of Techniques to Generate a Rater Decision Mechanism

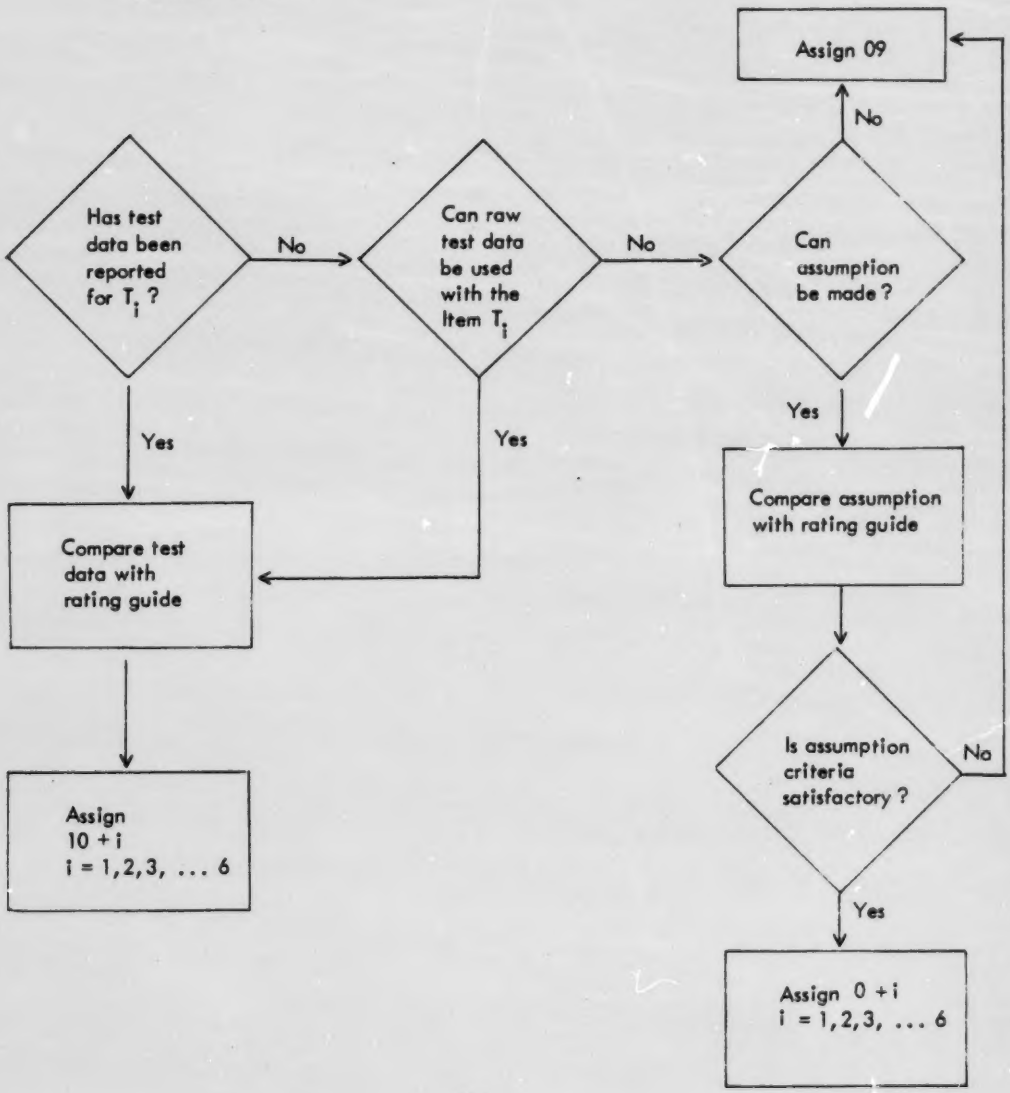
1. Development of a flow chart for application of the Standard Scales

2. Sampling of folders of subjects outside of the study to check on the applicability of the rater decision mechanism and flow chart

3. Detailed plans for the establishment of a rating manual to enhance rater reliability

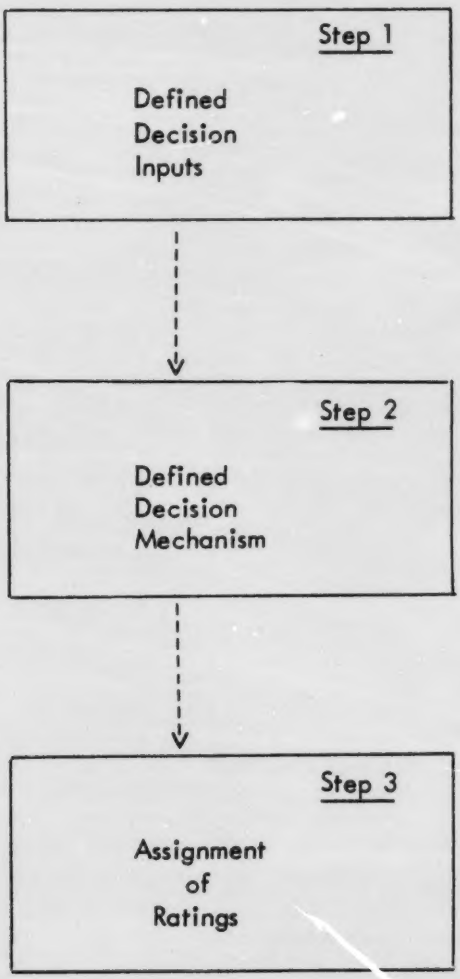
FLOW CHART FOR APPLICATION OF STANDARD SCALES IN THE RATER DECISION MECHANISM
Continued

Assume each item T_1, T_2, T_3 . . . . is being rated according to the Standard Scales.



FLOW CHART FOR APPLICATION OF STANDARD SCALES IN THE RATER DECISION MECHANISM

Steps in Rater Decision Mechanism



I. Development of Clearer Clinical Definitions of Learning Styles in Mathematics

1. Formulation of Learning Style I hypothesis

2. Formulation of Learning Style II hypothesis

J. Initiation of Statistical Procedures

1. Establishment of initial hypotheses

2. Discussions with statistical consultants regarding the initial steps in the statistical analyses once the data are fully coded

3. Initial coding of the data using the coding key and scales to establish inter-rater and intra-rater reliability

K. Establishment of Rating Manual for Case Histories (General Data) and the Coding of the Data (See Volume II, Section XI of this report

1. Expansion of the Coding Key to include WISC-R Full Scores (Verbal and Performance) plus WISC-R Subtest Scores, and scores for the Visual Motor Integration Tests

2. Decisions on where to obtain each item of data from the raw data available from the hospital records; which data override other data when more than one source is possible; and the precise meaning of each of the designated codes

3. Reading of all the folders to obtain and code the data for all the subjects

4. Confirmation of inter-rater and intra-rater reliability for the Case Histories (General Data) based on the Rater Manual

5. Consultations with statistical consultants on the questions and hypotheses to be investigated on this body of data taken as a separate entity

6. Initial computer runs of frequency distributions and analyses of the clinical populations

L. Establishment of Rating Manual for the Neuropsychological Data (See Volume II, Section XI of this report)

1. Expansion of the Coding Key to include two more items: Preferred Modality (Verbal, Part-for-Part Approach, or Perceptual, Gestalt, Configural Approach, or No Preference) and General Memory Deficits

2. Decisions of where to obtain each item of data from the various sections of the Neuropsychological Report; which data override other data when more than one source of data is available (for example, the Impressions Section which gives the neuropsychologist's own interpretations generally overrides the Raw Data/Findings, which have to be treated by us as implied rather than reported ratings); and the precise meaning of each of the designated codes.

3. Reading of all of the folders to obtain and code the data for all of the subjects. (This was a long, tedious job since some of the reports are 6 to 8 pages, single spaced, but an interesting and worthwhile experience to help establish further hypotheses to be tested.)

4. Confirmation of the inter-rater and intra-rater reliability for the Neuropsychological Data based on the Rating Manual

M. Establishment of the Rating Manual for the Neurological Data (See Volume II, Section XI of this report)

1. Expansion of the Coding Key to include another category

for hyperactivity measured by the Connor's Rating Scale

2. Decisions of where to obtain each item of data from the Neurologist's Report and how to deal with the fact that there were numerous styles of reporting by the ten to twelve different neurologists involved with the pool of subjects; which data override other data when more than one source is available; and the precise meaning of each of the designated codes

3. Reading of all of the folders to obtain and code the data for all of the subjects

4. Confirmation of the inter-rater and intra-rater reliability for the Neurological Data based on the Rating Manual

N. Establishment of the Rating Manual for the Mathematics Profile Data to take into account the grade level expectancies of the mathematics curriculum in schools. (See Volume II, Section XI of this report)

1. Further interpretations and clarifications of the Coding Key

2. Decision on grade level designations to go from the half-year point of one grade to the half-year point of the next

3. Decisions on where to obtain each item of data from the raw data recording forms and the mathematics specialist's reports; which data override other data when more than one source is available; and the precise meaning of each of the designated codes

4. Reading of all of the folders to obtain and code the data for all of the subjects

5. Confirmation of the inter-rater and intra-rater reliability for the Mathematics Profile Data based on the Rating Manual

Methods Used in the Study, Cont.

O. Initiation of Statistical Procedures

1. Punching of computer cards from the coding sheets

2. Accuracy checks on the recording of data and key-punching processes

3. Discussions with statistical consultants regarding the next stages in the statistical analyses

With all of this preliminary work, the data were ready for the statistical procedures and the testing of hypotheses. The following variables were used for the project.

Variables

Historical Data:

V3	Sex
V4	Month of Birth
V5	Date of Birth
V6	Year of Birth
V7	Month of Evaluation
V8	Date of Evaluation
V9	Year of Evaluation
V10	Chronological Age (Years) at Time of Evaluation
V11	Chronological Age (Months) at Time of Evaluation
V12	Grade in School
V13	Teacher-Reported Achievement in Mathematics
V14	Type of Standardized Math Test
V15	Level of Performance on Standardized Math Test
V16	Teacher-Reported Achievement in Reading Comprehension
V17	Teacher-Reported Achievement in Sight Vocabulary
V18	Teacher-Reported Achievement in Word Analysis
V19	Teacher-Reported Achievement in Spelling
V20	Teacher-Reported Attentional Functioning

Methods Used in the Study, Cont.

(Historical Data, Cont.)

V21 Teacher-Reported Perceptual Functioning
V22 Teacher-Reported Memory Function

Neuropsychological Variables:

V23 WISC-R Score: Verbal, Previcusly Administered
V24 WISC-R Score: Performance, Freviously Administered
V25 WISC-R Full Score: Previously Administered
V26 WISC-R Verbal Subtest: Information
V27 WISC-R Verbal Subtest: Similarities
V28 WISC-R Verbal Subtest: Arithmetic
V29 WISC-R Verbal Subtest: Vocabulary
V30 WISC-R Verbal Subtest: Comprehension
V31 WISC-R Verbal Subtest: Digit Span
V32 WISC-R Performance Subtest: Picture Completion
V33 WISC-R Performance Subtest: Picture Assembly
V34 WISC-R Performance Subtest: Block Design
V35 WISC-R Performance Subtest: Object Assembly
V36 WISC-R Performance Subtest: Coding
V37 Visual Motor Integration: Years
V38 Visual Motor Integration: Months
V39 Visual Motor Integration--Highest Item Score: Years
V40 Visual Motor Integration--Highest Item Score: Months
V43 Level of Ability
V44 Left Hemispheric Function
V45 Right Hemispheric Function
V46 Left Frontal Function
V47 Right Frontal Function
V48 Hemispheric Integration
V49 Integration of Components
V50 Organizational Functioning
V51 Attentional Functioning
V52 Sequencing Skills
V53 Output/Production Skills
V54 Speed of Response
V55 Language Functioning

Methods Used in the Study, Cont.

(Neuropsychological Variables, Cont.)

V56	Perseveration Findings
V57	Impulse Control
V58	Initiating Behavior Findings
V59	Sustaining Behavior Findings
V60	Inhibiting Behavior Findings
V61	Shifting Behavior Findings
V62	Developmental (Maturity) Findings
V63	Perceptual Memory
V64	Linguistic Memory
V65	Hyperactivity
V66	Preferred Modality
V67	General Memory Deficits

Neurological Variables:

V70	Left Hemispheric Function
V71	Right Hemispheric Function
V72	Left Frontal Function
V73	Right Frontal Function
V74	Hemispheric Integration
V75	Integration of Components
V76	Organizational Functioning
V77	Attentional Functioning
V78	Sequencing Skills
V79	Output/Production Skills
V80	Speed of Response Functioning
V81	Language Functioning
V82	Perseveration Findings
V83	Impulse Control
V84	Initiating Behavior Findings
V85	Sustaining Behavior Findings
V86	Inhibiting Behavior Findings
V87	Shifting Behavior Findings
V88	Developmental (Maturity) Findings
V89	Perceptual Memory
V90	Linguistic Memory

Methods Used in the Study, Cont.

(Neurological Variables, Cont.)

V91	Speed in Timed Motor Tests
V92	Classical Neurological Findings
V93	Hyperactivity
V94	Handedness
V95	Footedness
V96	Eyedness
V97	Hyperactivity According to Connor's Rating Scale

Mathematics Variables:

V100	Number Concepts: Summary
V101	Counting Forward
V102	Counting Backwards
V103	Skip Counting
V104	Concept of Quantity
V105	Proficiency with Discrete Models
V106	Proficiency with Continuous Models
V107	Place Value Concepts
V108	Order of Numbers
V109	Mathematics Symbolism: Summary
V110	Whole Numbers
V111	Reversals in Digits
V112	Reversals of Place Values
V113	Fractional Numbers
V114	Decimals
V115	Operational Signs
V116	Shapes
V117	Arithmetic Summary
V118	Addition Concepts
V119	Subtraction Concepts
V120	Multiplication Concepts
V121	Division Concepts
V122	Addition Facts
V123	Subtraction Facts
V124	Multiplication Facts

(Mathematics Variables, Cont.)

V125 Division Facts
V126 Compensatory Strategies in the Recall of Facts
V127 Addition Without Regrouping
V128 Addition With Regrouping
V129 Subtraction Without Regrouping
V130 Subtraction With Regrouping
V131 Multiplication With One-Digit Multiplier
V132 Multiplication With Two-Digit Multiplier
V133 Division With One-Digit Divisor
V134 Divison With Two-Digit Divisor
V135 Addition of Fractions With Same Denominators
V136 Addition of Fractions With Different Denominators
V137 Money Skills
V140 Subtraction of Fractions With Same Denominators
V141 Subtraction of Fractions With Different Denominators
V142 Multiplication of Fractions Without Cancellation
V143 Multiplication of Fractions With Cancellation
V144 Division of Fractions
V145 Addition of Decimals
V146 Subtraction of Decimals
V147 Multiplication of Decimals
V148 Division of Decimals
V149 Mental Arithmetic
V150 Estimation Skills
V151 Concept of Integers
V152 Procedures With Integers
V153 Algebra Skills
V154 Geometry Summary
V155 Figure/Ground Discrimination
V156 Appreciation of Detail
V157 Appreciation of Overall Configuration
V158 Organizational Skills in Geometry
V159 Geoboard Activities
V160 Perceptual Imagination/Judgment
V161 Three-Dimensional Figures
V162 Geometric Problem Solving

(Mathematics Variables, Cont.)

V163 Problem Solving -- Intuitions and Strategies: Summary
V164 Pictorial Graphs
V165 Applications to Real-Life Situations
V166 Analogies
V167 Sequences
V168 Intuitive Functions
V169 Word Problems
V170 Attributes of Sets: Creature Cards
V171 Logic
V172 Arithmetic Problem Solving
V173 Achievement at Grade Level
V174 Development (Maturity) Findings
V175 Overall "Math Sense"
V176 Sustaining Behavior
V177 Attention to Detail
V180 Anticipated Math Growth
V181 Memory Skills: Summary
V182 Memory for Arithmetic Facts
V183 Memory for Multi-Step Procedures
V184 Memory for Directions
V185 Memory for Pertinent Mathematical Data
V186 Visual Memory
V187 Self-Image
V189 Spontaneous Self-Structuring
V190 Initiation of Procedures
V191 Implementation of Procedures
V192 Production of Responses
V193 Organizational Skills
V194 Integration of Multi-Step Procedures
V195 Precision of Responses
V196 Attentional Functioning
V197 Ability to Learn New Skills/Procedures
V198 Need for Manipulative Materials
V199 Need for Reauditorization Techniques
V200 Impulsivity
V201 Perseveration
V202 Ability to Respond to Complexity

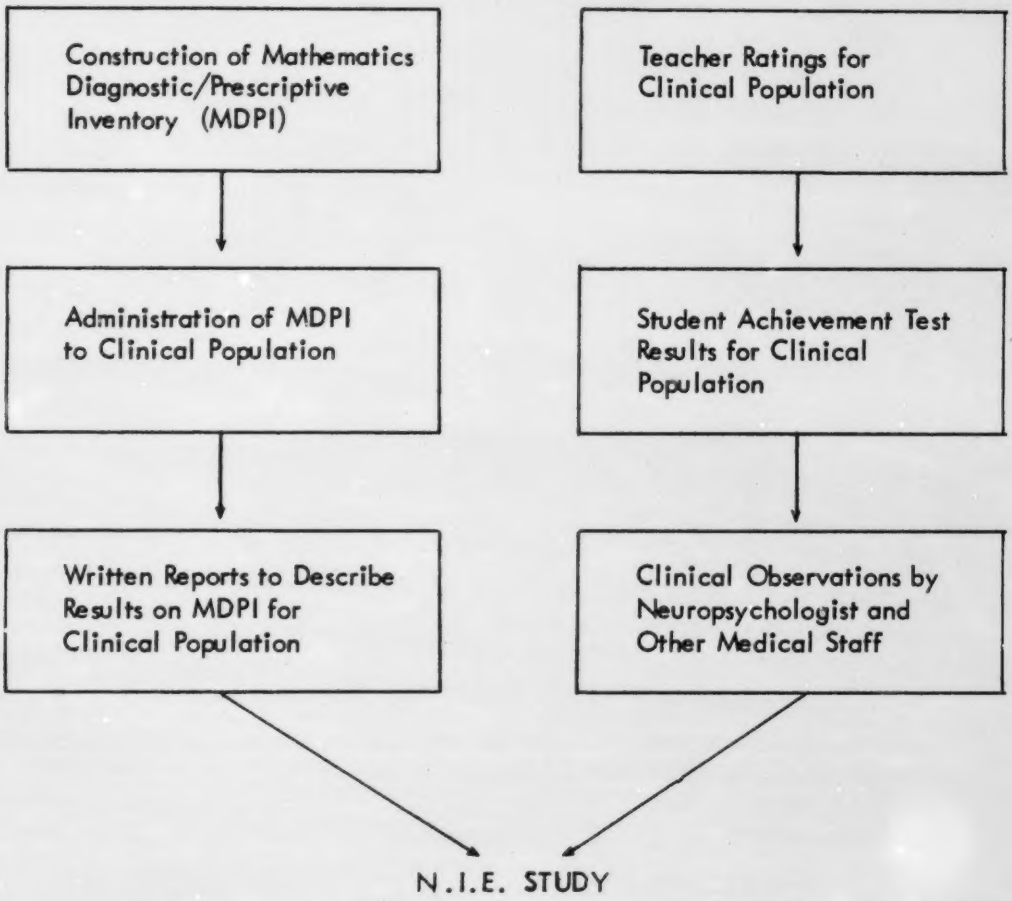
(Mathematics Variables, Cont.)

V203 Diversity in Approaches
V204 Flexibility in Approaches
V205 Recognition of Pertinent Math Data
V206 Mathematics Learning Style I Operative
V207 Mathematics Learning Style II Operative
V208 Mathematics Learning Style I Preferred
V209 Mathematics Learning Style II Preferred
V210 Mathematics Learning Style I Impaired
V211 Mathematics Learning Style II Impaired
V212 Behavioral Considerations
V213 Need for Extraneous Physical Activity
V214 Need for Extraneous Conversation
V215 Recommendations: Remediation Program
V216 Recommendations: In Regular Classroom
V217 Recommendations: Outside Regular Classroom
V218 Recommendations: One Individual (One-to-One Basis)
V219 Recommendations: Supportive Program
V220 Recommendations: Supportive Program in Regular Classroom
V221 Recommendations: Supportive Program Outside Regular Classroom
V222 Recommendations: Supportive Program on Individual (One-to-One Basis)
V223 Recommendations: Use of Discrete Models
V224 Recommendations: Use of Continuous Models
V225 Provision of Step-by-Step/"Recipe" Approaches
V226 Provision of Estimation "Preview" Approaches
V227 Provision of Strong External Structuring
V228 Relaxation of Time Constraints
V229 Recommendation: Development of Compensatory Strategies for Recall Deficits
V230 Provision of Calculator
V231 Recommendation: Experiential Math Program
V232 Recommendation: Math Program Determined by Life's Goals
V233 Recommendation: Close Follow-up Evaluation (6 mos. - 1 yr.)
V234 Language Interference
V235 Perceptual Judgment Problems

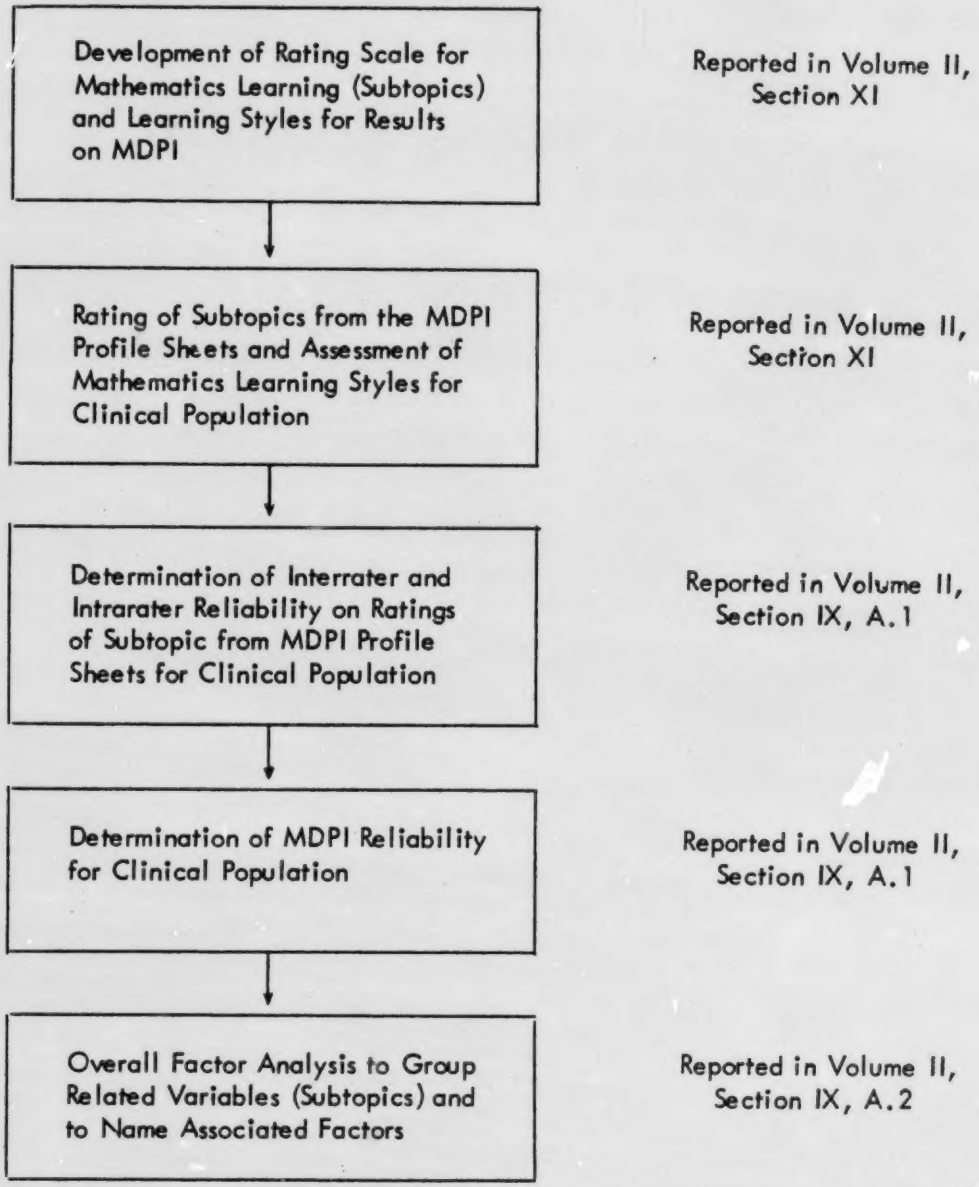
The detailed Coding Key for these variables is given in Volume II, Section X, followed by the detailed Rating Manual in Volume II, Section XI. Since a description of the statistical analyses of the research study is given in Volume II, Section IX, only a brief overview in chart form will be given here with cross-referencing to the particular sub-sections of Section IX. The topics are presented in the same order and treated with the same numbering as in the Overview of Research Design section of the grant proposal, for convenience of reference.

OVERVIEW OF RESEARCH DESIGN

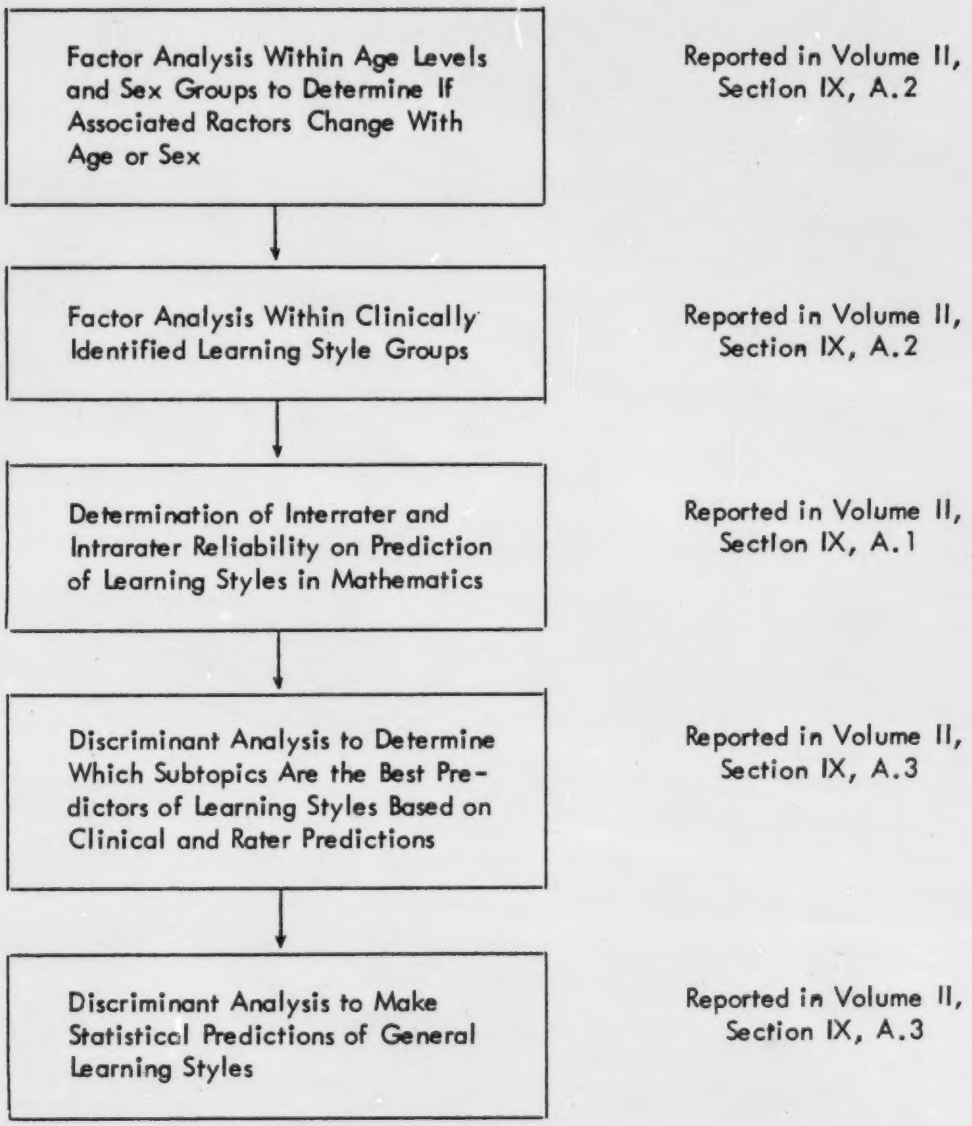
Work Completed in Preparation for Project Proposal


Construction of Mathematics Diagnostic/Prescriptive Inventory (MDPI)
Teacher Ratings for Clinical Population
Administration of MDPI to Clinical Population
Student Achievement Test Results for Clinical Population
Written Reports to Describe Results on MDPI for Clinical Population
Clinical Observations by Neuropsychologist and Other Medical Staff
N.I.E. STUDY

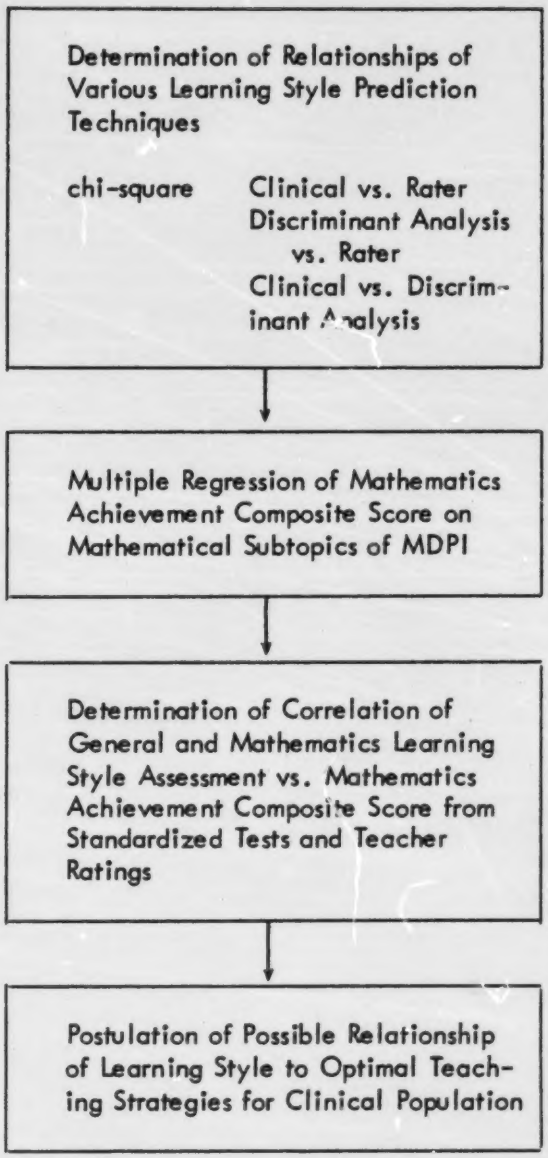
OVERVIEW OF RESEARCH DESIGN


Development of Rating Scale for Mathematics Learning (Subtopics) and Learning Styles for Results on MDPI
Reported in Volume II, Section XI
Rating of Subtopics from the MDPI Profile Sheets and Assessment of Mathematics Learning Styles for Clinical Population
Reported in Volume II, Section XI
Determination of Interrater and Intrarater Reliability on Ratings of Subtopic from MDPI Profile Sheets for Clinical Population
Reported in Volume II, Section IX, A.1
Determination of MDPI Reliability for Clinical Population
Reported in Volume II, Section IX, A.1
Overall Factor Analysis to Group Related Variables (Subtopics) and to Name Associated Factors
Reported in Volume II, Section IX, A.2

OVERVIEW OF RESEARCH DESIGN


Factor Analysis Within Age Levels and Sex Groups to Determine If Associated Factors Change With Age or Sex
Reported in Volume II, Section IX, A.2
Factor Analysis Within Clinically Identified Learning Style Groups
Reported in Volume II, Section IX, A.2
Determination of Interrater and Intrarater Reliability on Prediction of Learning Styles in Mathematics
Reported in Volume II, Section IX, A.1
Discriminant Analysis to Determine Which Subtopics Are the Best Predictors of Learning Styles Based on Clinical and Rater Predictions
Reported in Volume II, Section IX, A.3
Discriminant Analysis to Make Statistical Predictions of General Learning Styles
Reported in Volume II, Section IX, A.3

OVERVIEW OF RESEARCH DESIGN


Determination of Relationships of Various Learning Style Prediction Techniques
chi-square
Clinical vs. Rater
Discriminant Analysis vs. Rater
Clinical vs. Discriminant Analysis
Multiple Regression of Mathematics Achievement Composite Score on Mathematical Subtopics of MDPI
Determination of Correlation of General and Mathematics Learning Style Assessment vs. Mathematics Achievement Composite Score from Standardized Tests and Teacher Ratings
Postulation of Possible Relationship of Learning Style to Optimal Teaching Strategies for Clinical Population

Reported in Volume II, Section IX, A.3

Reported in Volume II, Section IX, A.3

Reported in Volume II, Section IX, A.3

Reported in Volume II, Section IX, A.3

SECTION III

RESULTS AND FINDINGS

- DEMOGRAPHIC FINDINGS
- TEACHER-REPORTED DELAYS IN VARIOUS AREAS
- CLINICALLY GENERATED DEMOGRAPHICAL FINDINGS
- NEUROPSYCHOLOGICAL FINDINGS CLEARLY REPORTED
- LEARNING STYLES IN MATHEMATICS
- FINDINGS GENERATED FROM RATING TECHNIQUES FOR NEUROLOGY, NEUROPSYCHOLOGY, AND MATHEMATICS LEARNING STYLES

SECTION III

RESULTS AND FINDINGS

Although findings are generally reported in terms of the results of statistical analyses, the "state of the art" of specific learning disabilities and learning styles in mathematics warrant the reporting of some of the characteristics that prevailed in the population under study. Specific conclusions from statistical analyses will follow these demographic findings.

Demographic Findings

- The population referred to the Learning Disabilities Clinic during the period under study was predominantly male (73.6%), which made it impossible to deal statistically with issues of sex differences.

- The modes of the grades in school for the population were Grades 3-6.

- Most subjects (55.9%) were reported as delayed with respect to mathematical achievement. However, this percentage is less than the delay reported in six other areas, as seen in the chart entitled Teacher-Reported Delays in Various Areas. Mathematical delay ranks seventh among the eight possible areas of delay reported.

- Very few individuals were reported to have had a standardized test in mathematics, particularly an individualized mathematical evaluation.

- Mathematical ability did not seem equally considered as a determiner of learning disabilities by teachers and parents. Reading skills were the prime consideration. Thus, prior to the evaluation done at the Children's Hospital Medical Center, mathematical skills were rarely subjected to careful analysis in and of themselves or as they relate to general cognitive profiles.

Teacher-Reported Delays in Various Areas

Attentional Functioning	74.0%
Spelling	62.8%
Reading Comprehension	60.9%
Word Analysis	59.6%
Perceptual Function	59.1%
Memory Skills	57.3%
Mathematics Achievement	*55.9%
Sight Vocabulary	42.7%

*Ranks 7th

Results and Findings, Cont.

- The distribution of cognitive ability as reflected by the WISC scores were close to a normal distribution, skewed slightly below average.

Clinically Generated Demographical Findings

In the neuropsychological reports, clear references were made to hemispheric findings in 112 out of the 220 subjects, as shown in the chart entitled Neuropsychological Findings Clearly Reported. The largest number of cases (81) indicated left hemispheric deficits, often specifically expressed as language difficulties. Few subjects (28) were described in terms of right hemispheric considerations. Those that were so described were generally described as deficient. Such imbalanced sorting according to neuropsychological differentiation may indicate to some extent the nature of the population under study, but may also reflect the greater difficulty in recognizing, defining and evaluating right hemispheric contributions to learning than left hemispheric skills.

Learning Styles in Mathematics

- As seen from the chart entitled Preferred Learning Style, more subjects preferred Learning Style I over Learning Style II (54.5% vs. 17.7%); 5.9% showed preference for neither learning style; and 21.8% had unclear data in determining preference.

- As seen from the chart entitled Impaired Learning Style, many subjects showed dual impairments; that is, impairments in both learning styles in mathematics.

  The data also indicated that more subjects who had impairments in Learning Style I had strength in Learning Style II (37 out of 171) than subjects who had impairments in Learning Style II who had strength in Learning Style I (12 out of 171). This indicates the need for Learning Style II teaching strategies to be identified and utilized for those students with that type of strength.

Neuropsychological Findings Clearly Reported

LEFT HEMISPHERIC DEFICITS	**N = 81**
LEFT HEMISPHERIC SKILLS INTACT	**N = 3**
RIGHT HEMISPHERIC DEFICITS	**N = 22**
RIGHT HEMISPHERIC SKILLS INTACT	**N = 6**

Total Number of Hemispheric Findings Clearly Reported — 112 out of 220.

Learning Styles in Mathematics

Clinically Assigned — (N = 220)

PREFERRED LEARNING STYLE

✧ Learning Style I	(N = 120)	54.5%
✧ Learning Style II	(N = 39)	17.7%
✧ Neither Preferred	(N = 13)	5.9%
✧ Preference Unclear from Available Data	(N = 48)	21.8%

Learning Style in Mathematics
Clinically Assigned — (N = 220)

IMPAIRED LEARNING STYLE

- ✧ **Learning Style I** **(N = 126)**
- ✧ **Learning Style II** **(N = 101)**
- ✧ **Neither Impaired** **(N = 31)**

NOTE: **MANY SUBJECTS HAVE DUAL IMPAIRMENTS; THAT IS, IMPAIRMENTS IN BOTH LEARNING STYLES IN MATHEMATICS.**

Findings Generated from Rating Techniques for Neurology, Neuropsychology, and Mathematical Learning Styles

- In the generating of a rater's manual for neurological and neuropsychological data, characteristics such as "Left Hemispheric Function" had to be rated on subjective descriptions by the clinician rather than on raw data which was clearly defined. More consistent reporting and assessment techniques need to be developed in order for the assignment of ratings to become more objective.

- Refined expectations for specific performance on the MDPI topics were generated by grade levels.

  - Existing criteria for achievement did not provide clear guidance for topic vs. grade/age expectations.

  - The convention of measuring grades from mid year of one year to the mid year of the next year (February Grade X to January X+1) proved to be a useful basis for assessment and evaluation criteria in mathematics. Mathematical abilities had to be rated on the basis of a range of expectation rather than on a single calendar point.

  - General "Rules of Thumb" for calculation skills were synthesized from more specific lists.

  - Non-arithmetic subtopics and items did not lend themselves easily to grade expectation analysis. This would suggest that process rather than actual solutions was more revealing in these topics.

- Each subject was rated on each math item. Thus an individual was "scored" as appropriate if the subject did not attempt the item as well as being "scored" as appropriate because of success with the item. This convention was adopted because not all items had been given to all subjects. However, this convention must

be carefully weighed when interpreting the statistical analyses, especially the factor analyses.

- Data for some students in some categories hence was not helpful. In future studies, statistical analyses must be conducted on subgroups so that data used is based on actual observations. Inference about appropriateness of non-administered items must be minimized.

SECTION IV

FINDINGS AND IMPLICATIONS OF THE STATISTICAL STUDY

- FINDINGS RELATED TO FACTOR ANALYSES
- IMPLICATIONS FROM FACTOR ANALYSES
- FINDINGS RELATED TO DISCRIMINANT ANALYSES
- IMPLICATIONS FROM DISCRIMINANT ANALYSES
- INDIVIDUAL COMPONENTS THAT APPEARED AS CONTRIBUTORS TO FOUR OR MORE PREDICTIVE CONSTELLATIONS
- GENERAL IMPLICATIONS FROM THE TOTALITY OF DISCRIMINANT ANALYSES
- HEURISTIC ISSUES

SECTION IV

FINDINGS AND IMPLICATIONS OF THE STATISTICAL STUDY

Findings Related to Factor Analyses

Factor analyses were used to determine relevant clusters of related subtopics on the MDPI with particular views to constellations hypothesized as Learning Style I and Learning Style II.

Three areas were studied:

(1) Factor analysis of the overall set of MDPI variables

(2) Factor analysis on subjects over nine years of age

(3) Factor analysis on selected MDPI items considered to be contributory to learning styles in mathematics

Since these factor analyses are discussed in detail in Volume II, Section IX, A.2, only summary listings will be given here.

Factor Analysis on MDPI Subtopics

Factor 1:

Decimal Notation
Fraction Operations (+, -, X, ÷)
Decimal Operations (+, -, X, ÷)
Concept of Integers
Procedures with Integers
Algebra Skills

Factor 2:

Number Concepts

Counting Forward
Counting Backwards
Skip Counting

(Factor 2, Cont.)

- Concept of Quantity
- Proficiency with Discrete Models
- Place Value Concepts
- Order of Numbers

Whole Number Symbolism

Arithmetic Summary

- Addition and Subtraction Concepts
- Addition and Subtraction Facts
- Compensatory Strategies in the Recall of Facts
- Addition without Regrouping
- Addition with Regrouping
- Subtraction without Regrouping

Applications to Real-Life Situations
Achievment at Grade Level
Developmental Findings
Sustaining Behaviors

Factor 3:

Mathematics Symbolism Summary

Arithmetic Skills

- Multiplication Concepts
- Multiplication Facts
- Multiplication with One-Digit Multiplier
- Multiplication with Two-Digit Multiplier
- Division Facts
- Division with One-Digit Divisor

Fractional Numerals

- Addition of Fractions with Same Denominator

Findings and Implications of the Statistical Study, Cont.

Factor 4:

Geometry Summary
Appreciation of Details
Appreciation of Overall Configuration
Organizational Skills in Geometry

Factor 5:

Multiplication Concepts
Multiplication Facts
Subtraction with Regrouping
Multiplication with One-Digit Multiplier
Problem Solving Summary
Sequences
Overall "Math Sense"

Factor Analysis on Subjects Greater Than 9 Years of Age

Factor 1:

Proficiency with Discrete Models
Subtraction Concepts
Addition without Regrouping
Addition with Regrouping
Subtraction without Regrouping

Factor 2:

Counting Forward
Counting Backwards
Multiplication Concepts
Divison Concepts
Multiplication with One-Digit Multiplier
Multiplication with Two-Digit Multiplier

Findings and Implications of the Statistical Study, Cont.

Factor 3:

Appreciation of Detail
Appreciation of Overall Configuration
Sequences
Math Learning Style II Impaired (Negative Factor Loading)

Factor 4:

Math Learning Style I Preferred (Negative Factor Loading)
Math Learning Style II Preferred

Factor 5:

Division Concepts
Multiplication with Two-Digit Multiplier
Divison with One-Digit Divisor
Divison with Two-Digit Divisor

Factor Analysis on Selected MDPI Items Considered to be Contributory to Learning Style in Mathematics

Factor 1:

Proficiency with Discrete Models
Proficiency with Continuous Models
Addition without Regrouping
Addition with Regrouping

Factor 2:

Division Concepts
Multiplication with Two-Digit Multipliers
Division with One-Digit Divisor

Findings and Implications of the Statistical Study, Cont.

Factor 3:

Multiplication Concepts
Subtraction with Regrouping
Multiplication with One-Digit Multiplier

Factor 4:

Math Learning Style I Preferred
Math Learning Style II Preferred (Negative Factor Loading)

Factor 5:

Appreciation of Detail
Appreciation of Overall Configuration
Math Learning Style II Impaired (Negative Factor Loading)

Implications from Factor Analyses

Factor analyses generally sorted subtopics on the basis of mathematical skills traditionally expected at particular grade levels. Learning style constellations were apparently preempted by achievement criteria. This would suggest that learning styles more strongly reflect qualitative aspects of performance versus success/non-success criteria on a specific math skill.

Findings Related to Discriminant Analyses

Discriminant analyses were used to determine the best predictors of

- Mathematics Learning Style I
- Mathematics Learning Style II
- Left Hemispheric Function with Respect to General Learning Style

- Right Hemispheric Function with Respect to General Learning Style

The predictors were selected from

- MDPI Subtopics
- MDPI Generalized Findings
- MDPI Subtopics and Findings

The following summary chart, entitled Levels of Prediction from Discriminant Analysis, indicates the level of prediction of a particular constellation generated.

Levels of Prediction from Discriminant Analysis

	PREDICTORS		
	MDPI Subtopics	MDPI Findings	MDPI Subtopics & Findings
NEUROPSYCHOLOGICAL DATA			
LEFT HEMISPHERIC FUNCTIONING	62.2%	66.2%	63.2%
RIGHT " "	— —	— —	— —
LEFT FRONTAL "	— —	69.2%	69.7%
RIGHT FRONTAL "	— —	71.6%	73.1%
NEUROLOGICAL DATA			
LEFT HEMISPHERIC FUNCTIONING	68.3%	67.8%	68.8%
RIGHT " "	61.8%	63.3%	65.8%

(Continued)

	PREDICTORS		
	MDPI Subtopics	MDPI Findings	MDPI Subtopics & Findings
MATH LEARNING STYLES			
STYLE I PREFERRED	79.2%	69.1%	80.3%
STYLE II "	78.7%	70.2%	81.5%
STYLE I IMPAIRED	80.0%	— —	— —
STYLE II "	77.7%	— —	— —

Implications from Discriminant Analyses

The best predictions were made of math learning styles. The strongest constellations for such predictions were obtained when considering both individual subtopics, quite objective ratings, as well as the MDPI-generated findings which included some aspect of subjective judgment.

Discriminant Analysis 3.2.1: To Predict Math Learning Style Preferred

The findings strongly support some of the components hypothesized as elements of Math Learning Styles I and II; specifically, Place Value, Appreciation of Detail, and Operations Signs.

The negative contributors are hypothesized in the Math Learning Style designations. Since Math Learning Style I and Math Learning Style II are mutually exclusive and account for all subjects, the negative contributors to Math Learning Style I would tend to suggest components of Math Learning Style II. The negative contributors hypothesized as Math Learning Style II components include Appreciation of Overall Configurations, Proficiency with Continuous Models, Shapes, Problem Solving Summary, and Real-Life Applications.

The factors with the higher coefficients tend to indicate the relative contributions of the given set of components. The relatively important factors in this set were Appreciation of Detail, Appreciation of Overall Configuration, and Place Value. Interestingly, these ratings emanate from less formal activities than those that are formal curriculum topics, and thus are not resultant from direct teaching.

Discriminant Analysis 3.2.2:
To Predict Math Learning Style II Preferred

These constellations include some of the hypothesized components of Math Learning Style I and Math Learning Style II. However, there were many confusing inclusions. Some contributors did not seem internally consistent, e.g. Math Symbolism was a positive contributors; while Whole Numbers and Decimals were negative contributors, but are included along with fractions in the category, Math Symbolism.

The contribution of the Appreciation of Detail versus the Appreciation of Overall Configuration is not as hypothesized and is not consistent with Discriminant Analysis 3.2.1. Similarly, use of discrete versus continuous models does not correspond with the hypothesis and Discriminant Analysis 3.2.1.

Discriminant Analysis 3.2.3:
To Predict Math Learning Style I Impaired

The role of arithmetic calculations in Math Learning Style I is highly confirmed. The small, but not necessarily expected, role of geometry no doubt indicates the modest need of Math Learning Style I to organize the geometry activities.

Discriminant Analysis 3.2.4:
To Predict Math Learning Style II Impaired

This analysis has some interesting combinations of positive and negative factors and seems to confirm the need for Learning Style II components in a variety of mathematical situations.

Discriminant Analysis 3.3.1:
To Predict Left Hemisphere - Neuropsychological

The components "Implementation of Procedures" is confirmed as a strong contributor to Math Learning Style I, (Discriminant Analysis 3.2.3). Thus, the correspondence between left hemisphere function and Math Learning Style I is confirmed. The behavioral components point to the sensitivity of left hemisphere function with its specific concern with details as reliant on attentional skills.

Lack of organizational skills may speak to the lack of appreciation of overall perspectives, rather than specific ordering of elements.

Discriminant Analyses 3.3.2 and 3.3.3:
To Predict Left and Right Frontal Functions - Neuropsychological

Although no hypotheses were posed for frontal functions, the constellations generated conform to expectation. The fact that mathematical activities can predict frontal function is heartening.

Discriminant Analysis 3.3.4:
To Predict Left Hemisphere Function - Neurological

In this constellation, the elements of both Mathematics Learning Styles I and II were intermingled among the positive and negative contributors.

Discriminant Analysis 3.3.5:
To Predict Right Hemisphere Function - Neurological

The positive contributors were related to organizational and attentional factors, while the negative factors spanned disparate subtopics, behavioral postures, and aspects of both Mathematics

Learning Styles I and II. This constellation seemed inconclusive. Again, it seems to be more difficult to predict right hemisphere function than the left.

Discriminant Analysis 3.4.1:
To Predict Left Hemisphere - Neuropsychological

The negative contributors particularly documented key elements of Math Learning Style II: Order of Numbers, Fractional Numbers, Shapes, Division Facts, and Money. The positive contributors were generally supportive of elements of Math Learning Style I, Mathematics Symbolism, Procedures with Integers. Since subtraction facts can be generated by additive techniques, their role as a positive contributor can be understood.

Discriminant Analysis 3.4.2:
To Predict Left Frontal - Neuropsychological

The elements included in this predictive constellation all rely heavily on precise consideration and implementation, particularly in the numerical realm. The elements Decimals, Subtraction Facts, Algebra Skills, Sequences, Organizational Skills, and Flexibility in Approaches are all consistent with Math Learning Style I, which is hypothesized as dependent generally on the left hemisphere.

The negative contributors are less consistent with anticipated findings, particularly Precision of Responses. Reversals in Digits and Visual Memory are expected to relate to Math Learning Style II, and thus Right Hemisphere Function so negative coefficients would be anticipated.

The inclusion of the variable, Algebra Skills, which would have been coded with many "false positives" may have contributed to the lack of consistency in this constellation.

Discriminant Analysis 3.4.3:
To Predict Right Frontal Function - Neuropsychological

Though this constellation is described as a strong predictor, the elements do not seem bound together by a particular theme or themes. Of interest is the suggestion of Math Learning Style I in terms of being "operative", "preferred", and "impaired", with "preferred" and "impaired" contributing negatively. This would seem contradictory at first. However, with less impairment and preference, the prediction for right function is enhanced. This would support the implication of the right hemisphere positively contributing.

Discriminant Analysis 3.4.4:
To Predict Left Hemisphere Function - Neurological

Several positive elements here, like Sequences, and the Need for Extraneous Conversation, and the Need for Manipulative Materials are consistent with Left Hemispheric Function. Furthermore, it would be expected that Reversals in Digits and Overall "Math Sense" would be negative factors.

Discriminant Analysis 3.4.5:
To Predict Right Hemispheric Function - Neurological

Although this constellation is only a "modest" predictor, it lends support to the definition of Math Learning Style II through the inclusion of Math Learning Style II Preferred, and the negative impact of multi-step calculations. The inclusion of memory components is most intriguing, particularly because of the elusive understanding of memory.

Discriminant Analysis 3.4.1:
To Predict Math Learning Style I Preferred

The heavy contribution of elements related to precision and precise production of solutions is most consistent with the proposed character of Math Learning Style I.

The negative factors suggest the need of input from Math Learning Style II to be an efficient problem solver. A strong Math Learning Style I does not seem sufficient.

Discriminant Analysis 3.5.2:
To Predict Math Learning Style II Preferred

The constellation generated is quite similar to that for Math Learning Style I Preferred, raising the question of how distinctly defined or rated each of these outcomes is.

Discriminant Analysis 3.6.1:
To Predict Math Learning Style I Preferred

The positive and negative factors not only strongly predict Math Learning Style I, they strongly support the hypothesized factors contributing to Math Learning Style I and Math Learning Style II.

The combination of subtopics and ratings yielded the strongest predictive power, suggesting that both objective measures and sensitized subjective judgments are required.

Discriminant Analysis 3.6.2:
To Predict Math Learning Style II Preferred

In many ways, the elements generated are more consistent with some proposed elements for Mathematics Learning Style I. Again, the clarity of distinction between preference for Math Learning Styles I and II should be raised.

Individual Components that Appeared as Contributors to Four or More Predictive Constellations

COMPONENT	PREDICTOR FOR
Shapes (V116)	Left Hemisphere Neuropsychological Left Hemisphere Neurological Math Learning Style I Preferred Math Learning Style II Preferred
Problem Solving Intuitions and Strategies (V163)	Right Frontal Neuropsychological Math Learning Style I Preferred Math Learning Style II Preferred Math Learning Style II Impaired
Visual Memory (V186)	Left Frontal Neuropsychological Right Frontal Neuropsychological Left Hemisphere Neurological Right Hemisphere Neurological
Precision of Responses (V195)	Left Frontal Neuropsychological Right Hemisphere Neurological Math Learning Style I Preferred Math Learning Style II Preferred
Impulsivity (V200)	Left Hemisphere Neuropsychological Right Frontal Neuropsychological Left Hemisphere Neurological Right Hemisphere Neuropsychological Math Learning Style I Preferred Math Learning Style II Preferred

(Individual Components that Appeared as Contributors to Four or More Predictive Constellations, Cont.)

COMPONENT	PREDICTOR FOR
Behavioral Considerations (V212)	Left Hemisphere Neuropsychological Left Frontal Neuropsychological Right Frontal Neuropsychological Left Hemisphere Neurological

It should be noted that in Volume II, Section IX, A.3.7.1, there is a complete summary table listing all MDPI variables and their involvement in discriminant functions for associated group variables.

General Implications from the Totality of Discriminant Analyses

- Conclusions from Discriminant Analyses using MDPI subtopics for predictor variables conformed more closely to hypotheses.
- "Findings" from the MDPI were less consistent internally and with proposed hypotheses than were the MDPI subtopics.
- Discriminant analyses revealed that characteristics which were primarily qualitative were more sensitive to the prediction of learning styles in mathematics and general cognitive learning profiles than quantitative aspects.
- Both positive and negative contributors are required to make predictions about mathematical learning styles and general cognitive profiles. Failure to perform was often as valuable a predictor as successful solutions.

Findings and Implications of the Statistical Study, Cont.

(General Implications from the Totality of Discriminant Analyses, Cont.)

- Of the 18 discriminant analyses reported, 11 provided partial or full support of the proposed hypotheses regarding mathematical learning styles and the prediction of general cognitive profiles from mathematical data. Seven seemed inconclusive due to the variety of elements included.

- A few factors recurred in many predictive constellations. Of interest is that they were either non-formal math activities or behavioral postures, suggesting that these "non-learned" characteristics are most salient in reflecting mathematical and cognitive profiles.

- Although individual elements of predictive constellations are intricately interdependent, a scanning of all the elements emerging in the discriminant analyses or the strongly contributing elements highly documents the proposed hypotheses regarding two distinct mathematical learning styles. The predictive power of mathematical findings for more general characterizations was also supported in a general fashion.

Heuristic Issues

As an exploratory study, the results of this research are most encouraging. The initial hypotheses of math learning styles were supported and the extendability of math learning styles to predict more general cognitive profiles was promising.

The particular components of the mathematical learning styles need to be more clearly scrutinized and specified. To do so, requires the administration of the MDPI to a more general, carefully selected population representing a full range of cognitive ability, cognitive styles, and mathematical protocols. Further statistical analysis should be applied to subgroups rather than total population to avoid "false positives" that might emerge from differences in expectations according to age.

Findings and Implications of the Statistical Study, Cont.

The specific predictive power of mathematical learning styles to general cognitive profiles requires a more careful assessment and definition of the distinct cognitive profiles with their specific lateralizing characteristics delineated. A more balanced distribution of such defined cognitive profiles should then be incorporated into the total and subgroup populations.

The most exciting result of this research is the direct applicability it offers for the development of better teaching/learning experiences. The hypotheses for distinct mathematical learning styles warrant distinct and differentiated approaches to specific math topics. The exact differentiation may be suggested by the elements in each learning style, with special emphasis on the qualitative differences in processing. The following are examples of how these findings can be incorporated and utilized in the classroom.

SECTION V

PRESCRIPTIVE TEACHING

- FORMULATING EFFECTIVE TEACHING STRATEGIES
- EXAMPLES OF PRESCRIPTIVE TEACHING USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

SECTION V

PRESCRIPTIVE TEACHING

Formulating Effective Teaching Styles

Once hypothesized, learning styles in mathematics contribute directly to the choice of effective teaching strategies. In general terms, the recognition of differences in learning style mandates that a variety of models (discrete/continuous, verbal/perceptual) and a variety of approaches (carefully sequenced deductive procedures or highly inductive presentations) must be available in regular classrooms. For an individual having difficulty learning mathematics, a diagnosis of a specific learning style may suggest modification of expectations, topics, or approaches.

For the learning disabled child, it is particularly important to assess both the apparently preferred learning style and the apparently impaired learning style. If the deficits appear in the non-preferred style, then mathematics topics reinterpreted and presented in terms of the preferred modality and approach can circumvent deficiencies. When the preferred learning style is itself marked by deficits, supportive techniques become as important as remedial strategies. The impairments suggest that appropriate use of the individual's style rather than choice of style or topic is the prime consideration. The progress of an individual in this situation must be carefully monitored to insure appropriate use of the learner's skills.

For most people, successful mathematical experience is predicated upon successful harmony of both learning styles. The contribution of one learning style may be dominant. Both complementary learning styles must be exercised and implemented to teach as many mathematical skills as the individual can learn and to increase the quality of the learning.

Mathematics is itself the orchestration of a variety of diverse topics in a multitude of situations. The effective learning of mathematics requires abilities, skills, and learning styles blended in melodious concert.

The mere definition of a specific learning style in mathematics or a more general cognitive learning style is interesting in and of itself. However, it becomes more powerful and helpful as it is used to design educational programs and strategies. By appealing to a preferred learning

style, the opportunity to maximize mathematical learning is present.

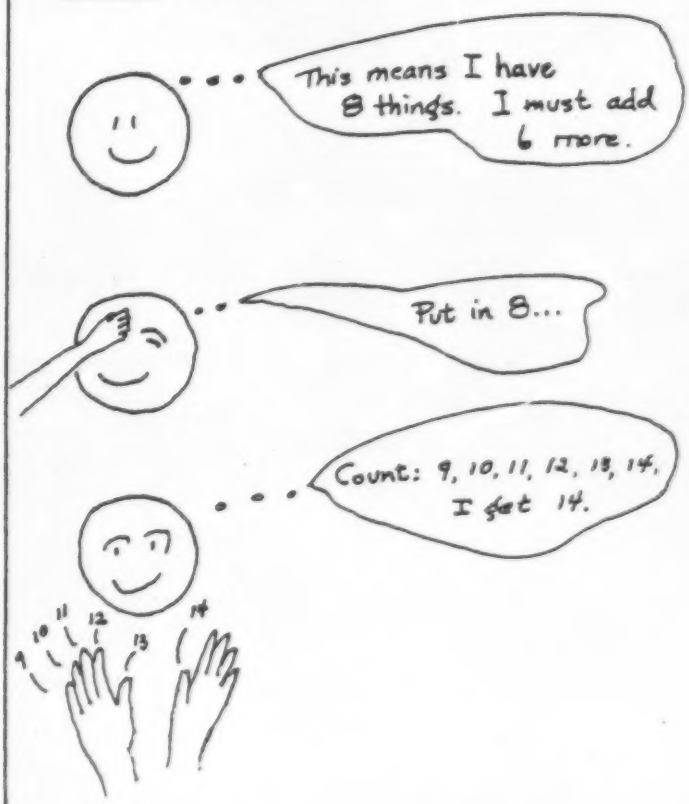
To help translate differentiated learning styles into differentiated teaching strategies, some typical mathematical topics have been developed in two distinct fashions. One approach emphasizes Mathematics Learning Style I. Another approach has been presented to treat the same topics from a Mathematics Learning Style II point of view.

PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in the Recall of Upper Decade Addition Facts
Basic Level

MATHEMATICS LEARNING STYLE I

Example: 8 + 6 = ?



Addition is seen as "counting on."

Sometimes students actually cup their hand and tap themselves on the forehead to vividly portray, "Put in 8 all at once."

Students count on 6 more. Often children use their fingers to count out and record the second set.

When students count only the second set (rather than both sets) on their fingers, their fingers don't run out.

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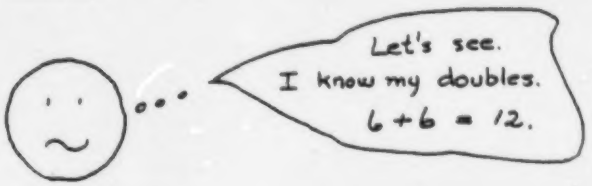
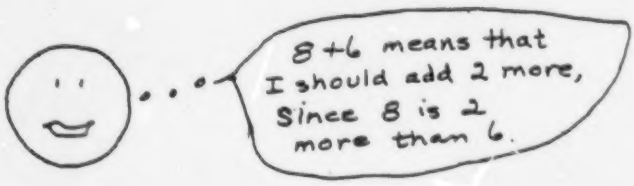
TOPIC: Compensatory Strategies for Deficits in the Recall of Upper Decade Addition Facts
Basic Level

MATHEMATICS LEARNING STYLE II

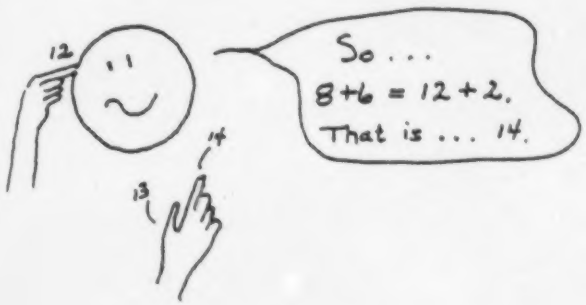
Example: 8 + 6 = ?



Students use a related arithmetic fact that involves one of the numbers. Students usually know doubles.



If the smaller number is taken, then it's always a matter of adding on to the doubles.



The two facts are assimilated simultaneously to yield a solution.

Another possibility would have been to take 8 + 8 and then subtract 2.

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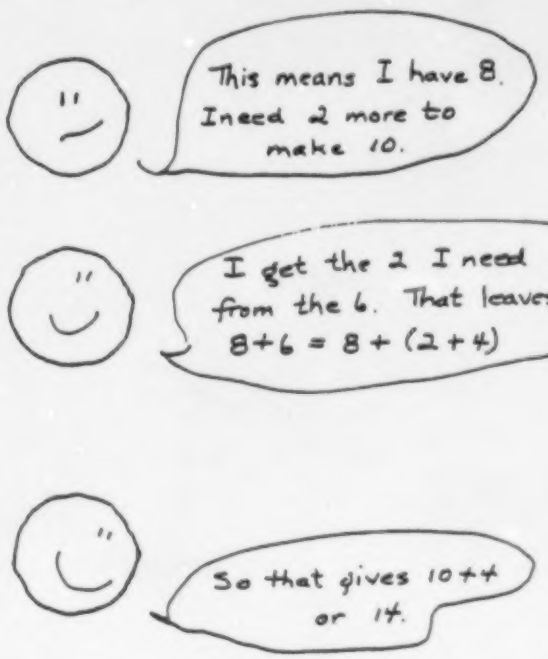
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in the Recall of Upper Decade Addition Facts
Intermediate Level

MATHEMATICS LEARNING STYLE I

Example: 8 + 6 = ?



The student "counts on" to 10, knowing missing addends for 10 is the vital step here. In this case, the student thinks: 8 + ? = 10.

The 6 is thought of as 2 + 4, since the 2 is what is needed to "build" the 10.

Essentially the 2 gets added to the 8:

$$\begin{aligned} 8 + 6 &= 8 + (2 + 4) \\ &= (8 + 2) + 4 \\ &= 10 + 4 \\ &= 14. \end{aligned}$$

As an alternative method, the student could have started with the 6 and built it up to 10, taking 4 from the 8. $(4 + 4) + 6 = 4 + (4 + 6) = 4 + 10 = 14$. However, it is always most efficient to start with the larger of the two addends.

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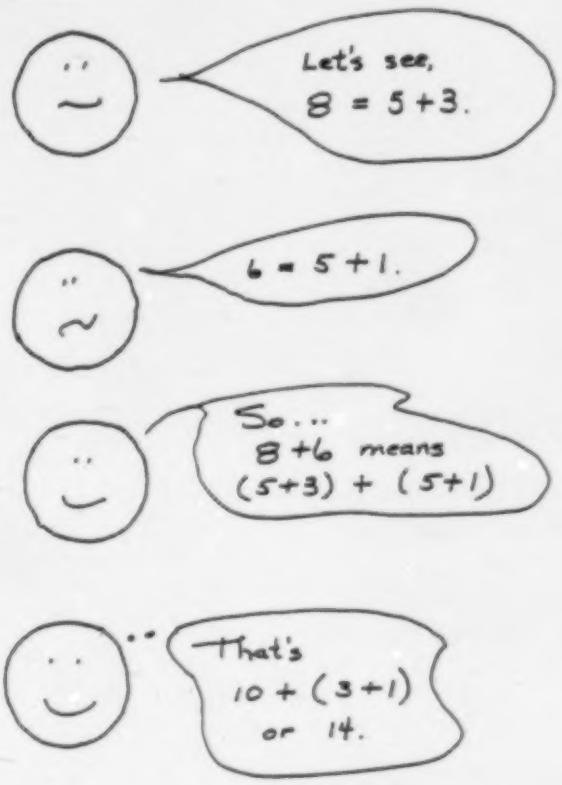
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in the Recall of Upper Decade Addition Facts
Intermediate Level

MATHEMATICS LEARNING STYLE II

Example: 8 + 6 = ?



Students use a relevant "5 plus fact." This is often immediate, since finger portrayal of a number encourages the form 5 plus. . .

Another 5 plus fact . . .

The two 5 plus facts can be put together.

The verbal thinking precludes the formal symbolism using parentheses. The abstraction of that thinking using symbols makes the process seem very understandable.

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PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategies for Deficits in the Recall of Upper Decade Subtraction Facts
Basic Level

MATHEMATICS LEARNING STYLE I

Example: 15 - 8

The student converts the subtraction problem to an addition problem--a missing addend situasituation.

He uses his fingers to count on . . .

Each step in the thinking process proceeds directly from the previous one.

Another Method:
Sometimes students start at 8, count up 2 to get 10 and then 5 more to get 15. They really counted up 7 in all. 8 + 7 = 15 or 15 - 8 = 7.

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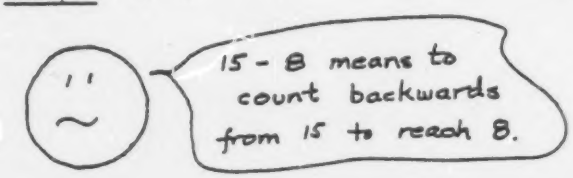
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

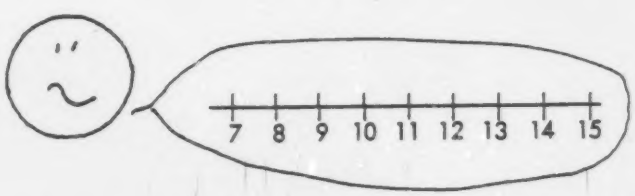
TOPIC: Compensatory Strategies for Deficits in the Recall of Upper Decade Subtraction Facts
Basic Level

MATHEMATICS LEARNING STYLE II

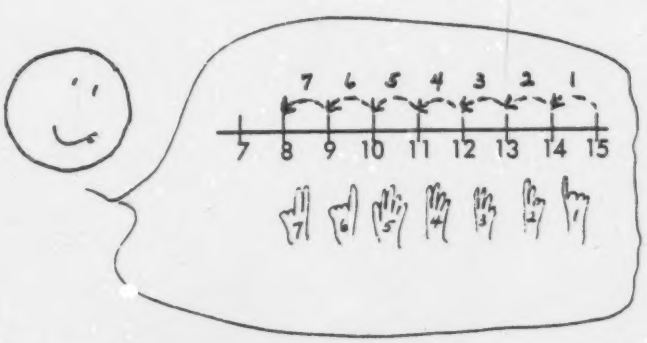
Example: 15 - 8



Subtraction is seen as counting backwards.



Students often picture a number line in their heads.



Students march backwards on the number line, keeping track of their hops by using their fingers.

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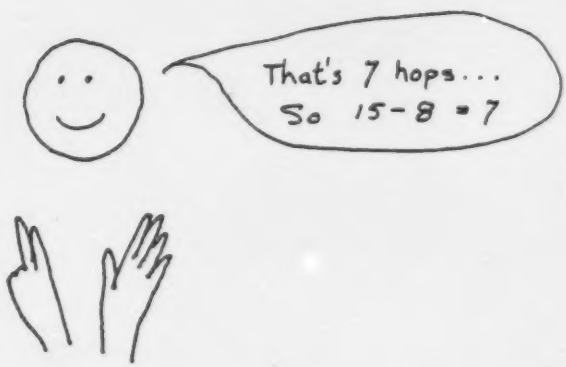
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategies for Deficits in the Recall of Upper Decade Subtraction Facts
Basic Level (Cont'd.)

MATHEMATICS LEARNING STYLE II

Example: 15 - 8 (Cont'd.)



Other Methods

Sometimes students will start at 15, count back 5 to get 10, and then 3 more to get 7. They really count back 8 in all. 15 - 8 = 7.

Also, they may think of 15 as 10 + 5 and subtract 3 more (8 in all) to give 10 - 3 or 7.

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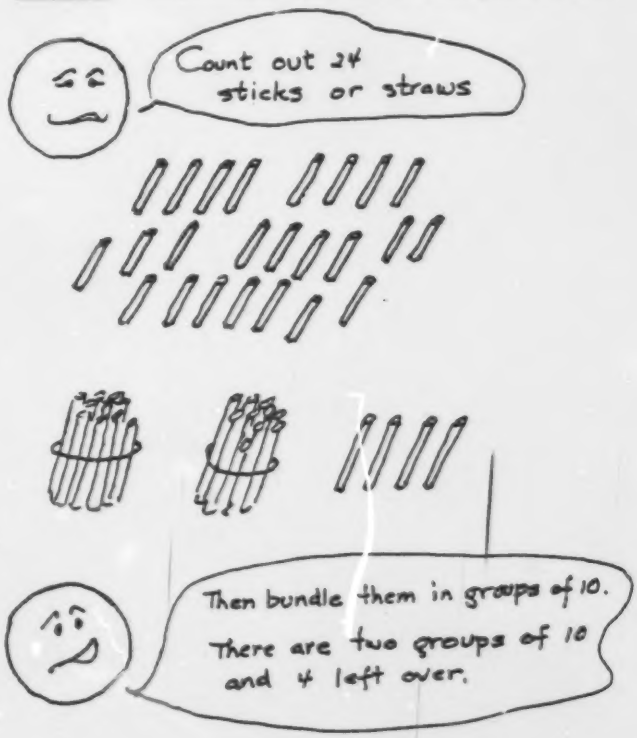
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Place Value
Basic Level

MATHEMATICS LEARNING STYLE I

Example: Demonstrate 24 in terms of its place value.



The student starts with a set of ones by counting out by ones a set of straws or sticks amounting to 24.

Groups of ten are then bundled together.

24 is seen as 2 bundles of 10 and 4 ones.

Any kind of counters (beads, seashells, chips, or cubes) can be used in a similar way. It is advisable that all of the objects be the same color and size. Students can bundle groups of 10 by placing sets of 10 objects in small paper cups. Again, 24 would be seen as 2 tens and 4 ones.

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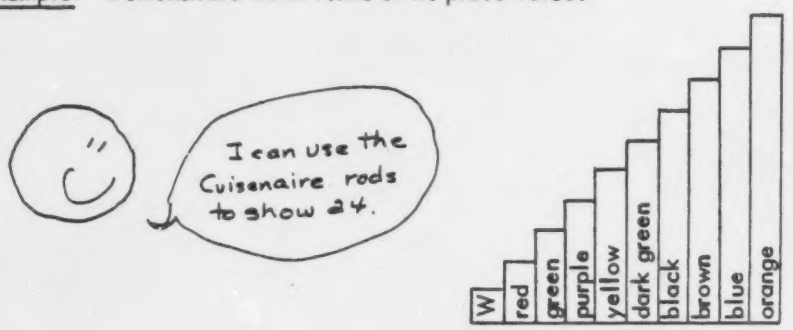
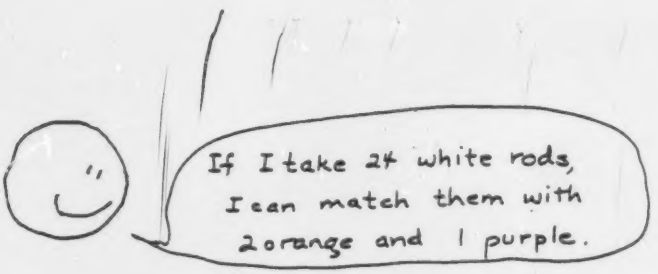
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

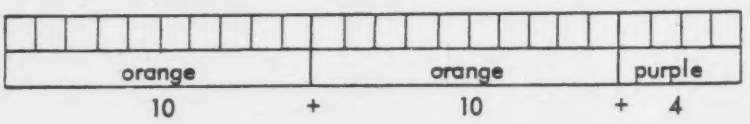
TOPIC: Place Value
Basic Level

MATHEMATICS LEARNING STYLE II

Example: Demonstrate 24 in terms of its place value.







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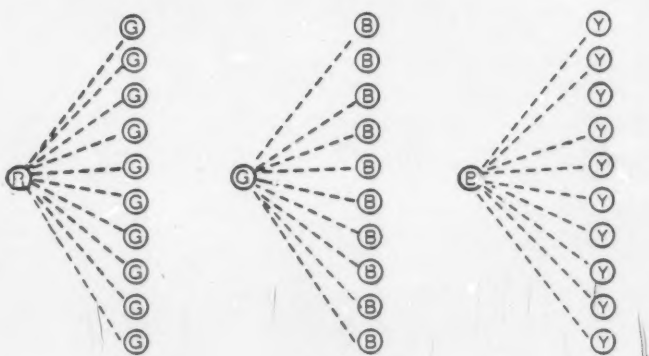
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Place Value
Intermediate Level

MATHEMATICS LEARNING STYLE I

Example: Demonstrate 24 in terms of its place value.

Chip Trading in Ten-For-One Country



In the Chip Trading Model, colored chips are used like coins. In Ten-For-One Country, the trading is based on tens, where 10 chips of one color are traded for 1 chip of the next higher valued color.

Neither the color nor physical size of the chips suggest their relative value. The order of the colors from least valuable (yellow) to most valuable (red) must be remembered. The trade of ten-to-one gives the place values 1, 10, 100, and 1000.

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TOPIC: Place Value
Intermediate Level (Cont'd.)

MATHEMATICS LEARNING STYLE I

Example: Demonstrate 24 in terms of its place value (Cont'd.)

I pretend that I am in a foreign land, where colored chips are used as money. In Ten-For-One Country, the exchange is set as:

10 yellow chips can be traded for one blue chip.

10 blue chips can be traded for 1 green chip.

10 green chips can be traded for 1 red chip.

10x10x10	10x10	10	1
R	G	B	Y
1000	100	10	1

The value of each colored chip in terms of yellow chips is shown here on the Chip Till.

For 24, I take 24 yellows and trade 10 yellows for 1 blue, and then 10 more yellows for a second blue. There are 4 yellows left over.

1000	100	10	1
R	G	B	Y
		O O	O O O O

Hence 24 can be seen as 2 tens and 4 ones.

My fully traded collection is 2 blues and 4 yellows.

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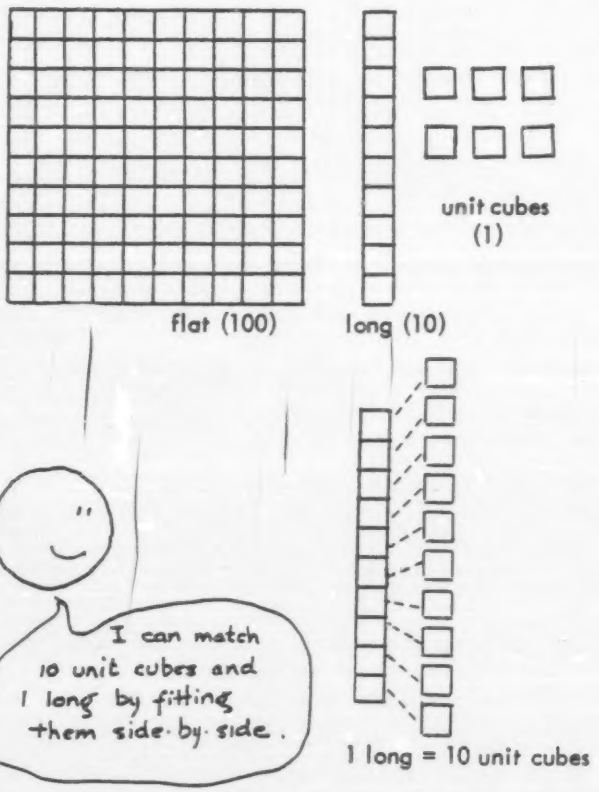
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Place Value
Intermediate Level

MATHEMATICS LEARNING STYLE II

Example: Demonstrate 24 in its place value.

A Continuous Model for Place Value



In the Dienes' Blocks, Powers of Ten (Number Blocks) Model, the actual sizes of the pieces of wood suggest their values. Students can actually match the pieces to find that:

10 unit cubes = 1 long
10 longs = 1 flat

The "trades" need not be remembered, as the relative values of the pieces can be physically measured.

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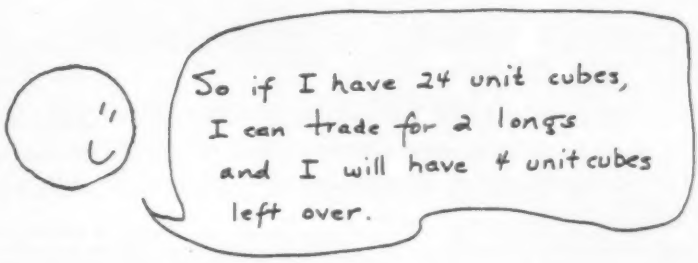
TOPIC: Place Value
Intermediate Level (Cont'd.)

MATHEMATICS LEARNING STYLE II

Example: Demonstrate 24 in its place value. (Cont'd.)



24 = 2 tens + 4 ones.

Students who prefer Learning Style II enjoy doing their work by measuring (matching) rather than counting. They also like the fact that you can pick up 100 (a flat) or 10 (a long) all at once.

Students who have deficiencies in Learning Style II sometimes have difficulty with this model, as they try to count the value of a "long" by counting the grid lines to obtain a value of 9, or the grid lines plus the end markings to obtain a value of 11. They do not see the 10 spaces on the long.

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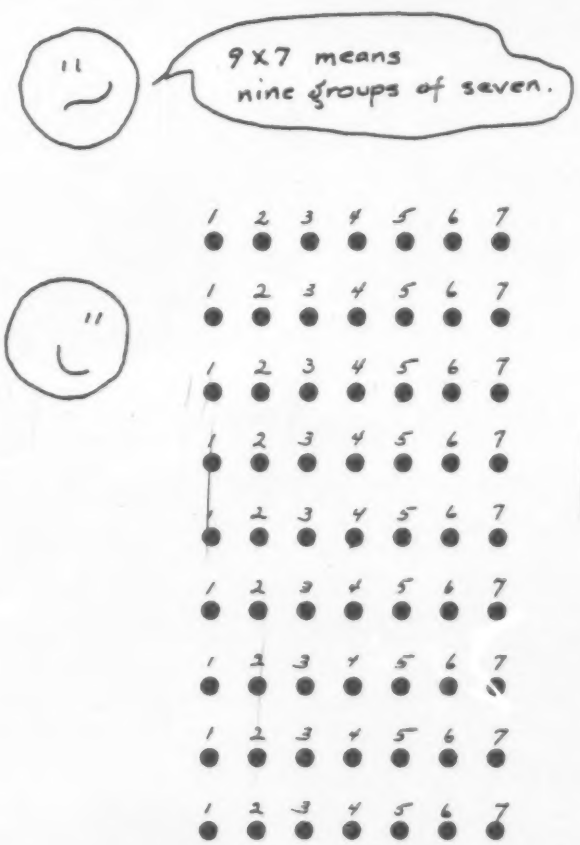
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in the Recall of Multiplication Facts
Basic Level

MATHEMATICS LEARNING STYLE I

Example: 9 x 7



Multiplication is seen as repeated addition of sets.

Students demonstrate the groups with sets of discrete objects. Seven is counted out each time.

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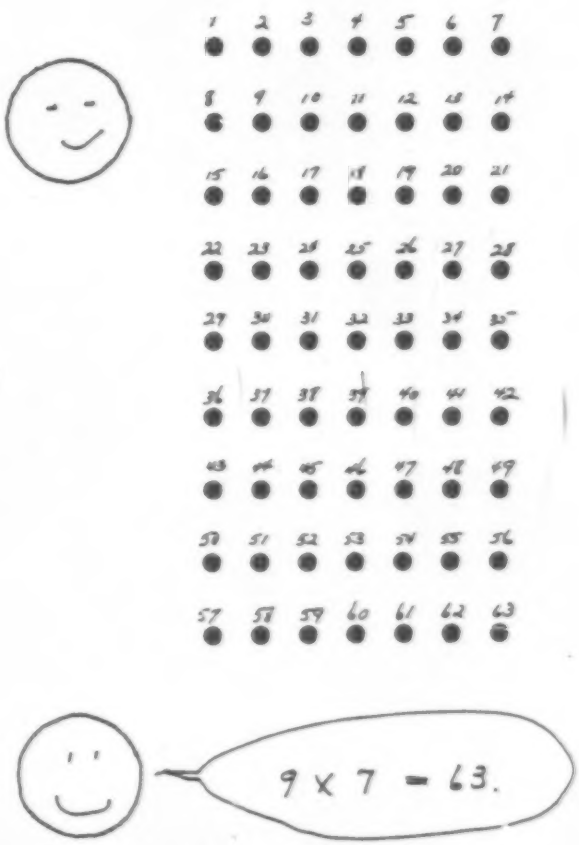
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in Recall of Multiplication Facts
Basic Level (Cont'd.)

MATHEMATICS LEARNING STYLE I

Example: 9 x 7 (Cont'd.)



The student counts up by ones the total number in the group.

A laborious process. With patience, organization and precision, it yields correct solutions. A cue that Math Learning Style I is being used is that the student gives an answer such as "9 x 7 = 61" indicating an error in counting.

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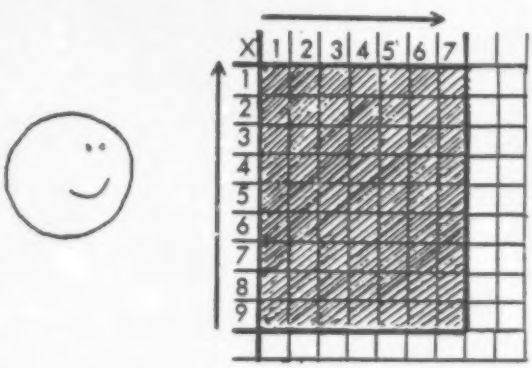
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in Recall of Multiplication Facts
Basic Level

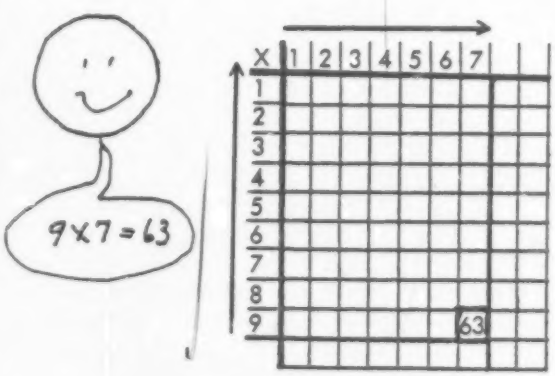
MATHEMATICS LEARNING STYLE II

Example: 9 x 7 =



The student uses the physical arrangement of the multiplication table to help.

The multiplication fact is interpreted as an area with given dimensions, 9 by 7.



The actual number of squares in the area of the rectangle corresponds to the numeral written in its lower right-hand corner.

Students often use a blank piece of graph paper to help cue the physical arrangements suggested by the multiplication problem.

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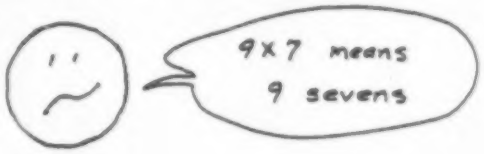
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

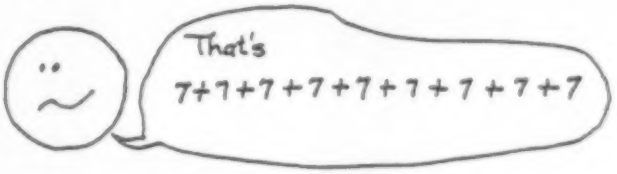
TOPIC: Compensatory Strategy for Deficits in Recall of Multiplication Facts
Intermediate Level

MATHEMATICS LEARNING STYLE I

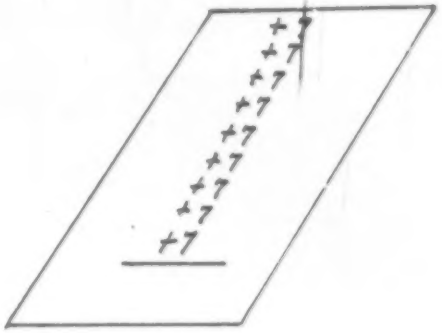
Example: 9 x 7



Multiplication is interpreted as repeated addition.



Students use numerals instead of actual elements of sets.



The student records the problem. To do the addition, students often return to their fingers.

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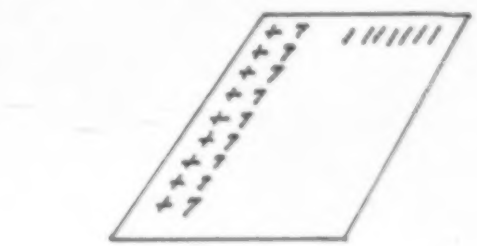
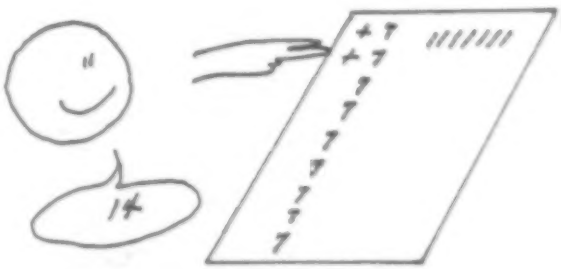
TOPIC: Compensatory Strategy for Deficits in Recall of Multiplication Facts
Intermediate Level

MATHEMATICS LEARNING STYLE I

Example: 9 x 7 (Cont'd.)



An interesting shortcut which organizes the counting is to draw a set of 7 slashes. Then the student counts off the set 9 times, using his fingers to record how many times the set has been counted.



....and so on to 63.

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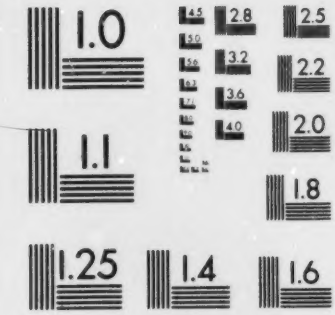


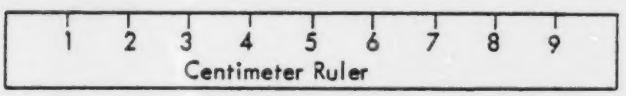
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

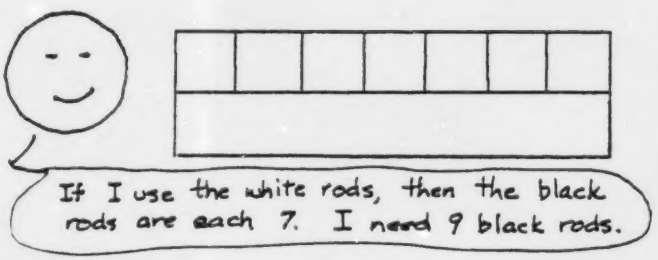
TOPIC: Compensatory Strategy for Deficits in the Recall of Multiplication Facts
Intermediate Level

MATHEMATICS LEARNING STYLE II

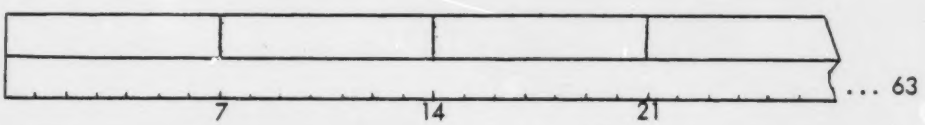
Example: 9 x 7

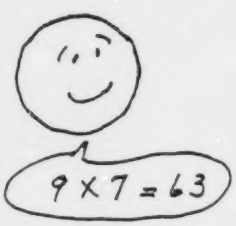


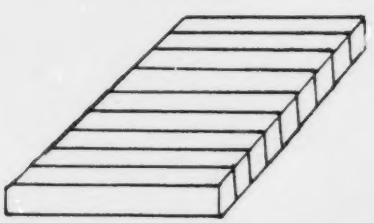
Students use a meter stick and Cuisenaire rods.



The Cuisenaire rods can be placed against the meter stick. The ends of the rods come at the multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63.







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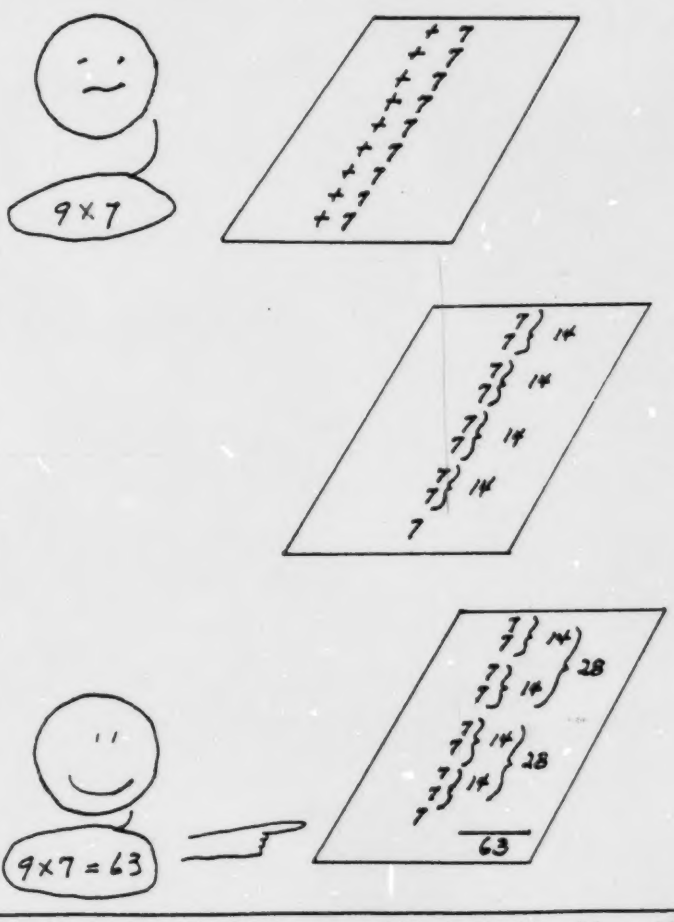
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in Recall of Multiplication Facts
Advanced Level

MATHEMATICS LEARNING STYLE I

Example: 9 x 7



The student shortens the counting technique by using familiar facts--often doubles.

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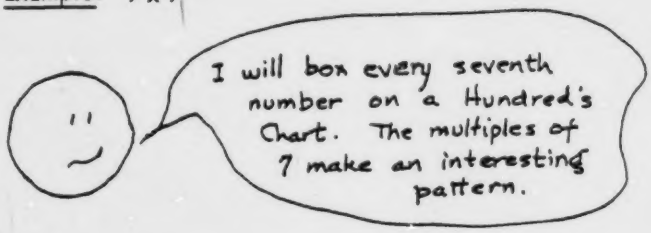
TOPIC: Compensatory Strategies for Deficits in Recall of Multiplication Facts
Advanced Level

MATHEMATICS LEARNING STYLE II

Example: 9 x 7



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The patterns in the number chart for multiples of 7 are observed.

The pattern for the multiples of 7 goes: move to the left 3 and go down 1." Using that pattern, other multiples of 7 can be generated.

This pattern corresponds to 7 being 10 - 3, (going down one row adds 10; moving to the left 3 subtracts 3.

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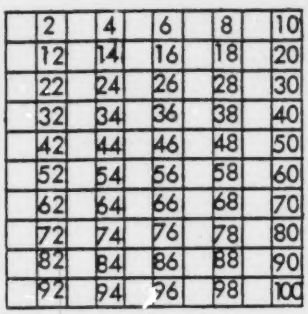
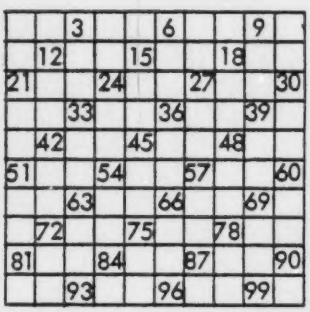
TOPIC: Compensatory Strategies for Deficits in the Recall of Multiplication Facts
Advanced Level

MATHEMATICS LEARNING STYLE II

General Techniques: The patterns for the multiples of other numbers also make interesting and artistic patterns on a Hundred's Chart. The Hundred's Chart gives a visual overview of the sets of numbers.



2



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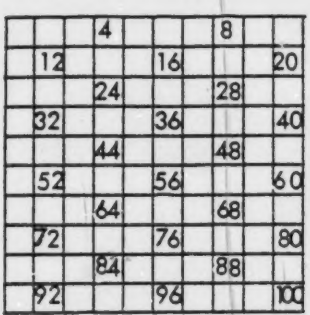
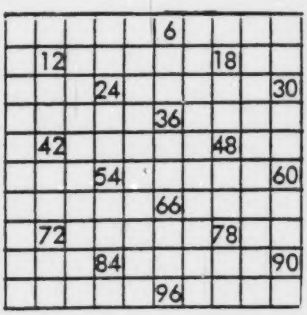


4



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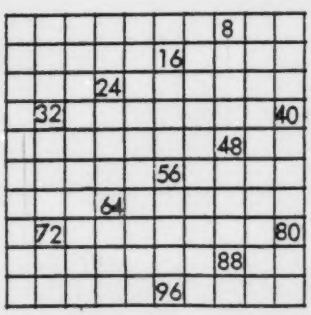
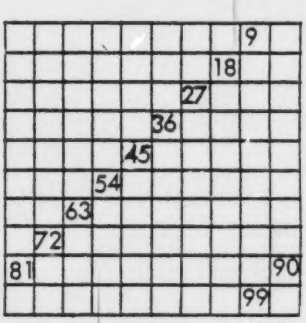


8



9

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PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategies for Deficits in the Recall of Multiplication Facts
Advanced Level

MATHEMATICS LEARNING STYLE II

General Techniques: Listing the sets of multiples and observing the one's digits also provides an overview by way of numerical patterns, for example:

Multiples of 2	Multiples of 8
0	0
2	8
4	16
6	24
8	32
10	40
12	48
14	56
16	64
18	72
20	80

The repeated pattern of 0, 2, 4, 6, 8 in the one's digits for multiples of 2 is reversed for the multiples of 8. (Note that 2 + 8 = 10. Adding 8 is like adding 10 and subtracting 2).

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TOPIC: Compensatory Strategies in the Recall of Multiplication Facts
Advanced Level (Cont'd.)

MATHEMATICS LEARNING STYLE II

Multiples of 4	Multiples of 6
0	0
4	6
8	12
12	18
16	24
20	30
24	36
28	42
32	48
36	54
40	60

The repeated pattern 0, 4, 8, 2, 6 in the one's digits for the multiples of 4 is reversed in the multiples of 6. (Note that $4 + 6 = 10$. Adding 6 is like adding 10 and subtracting 4).

Students might also relate the sets of multiples for 3 and 7 in a similar way.

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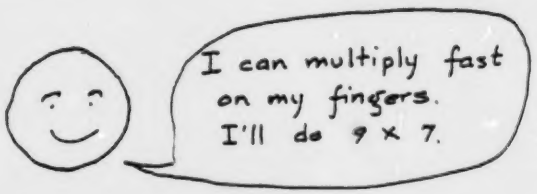
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Compensatory Strategy for Deficits in the Recall of Multiplication Facts
Advanced Level

MATHEMATICS LEARNING STYLE:
Finger Multiplication Method

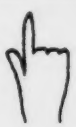
Example: 9 x 7



Left Hand

For 9: On my left hand, I put 4 fingers up and fold one finger down.

Right Hand

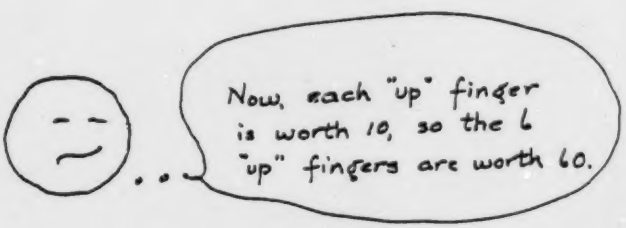


For 7: On my right hand, I put 2 fingers up and fold three fingers down.

The multiplication facts beyond 5 are the ones which give students the most difficulty: 7 x 6, 8 x 7, 9 x 6, 9 x 7, etc. They can be done with the following finger method.

Since a number larger than 5 has to be shown on just one hand, the number of fingers held up is the excess beyond 5.
$9 = 5 + \underline{4}$ → 4 fingers up
$7 = 5 + \underline{2}$ → 2 fingers up

There are 4 fingers up on the left hand and 2 fingers up on the right hand.



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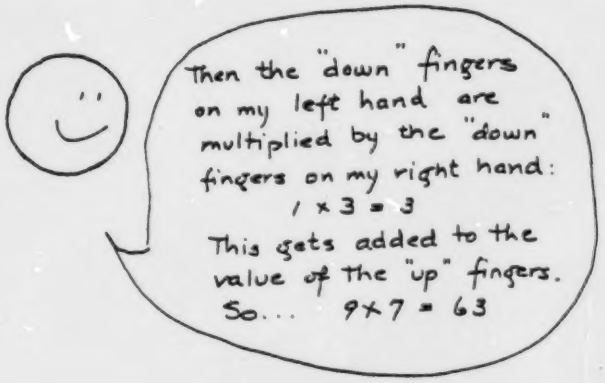
TOPIC: Compensatory Strategy for Deficits in the Recall of Multiplication Facts
Advance Level (Cont'd.)

MATHEMATICS LEARNING STYLE
Finger Multiplication Method

Example: 9 x 7 (Cont'd.)


Then the "down" fingers on my left hand are multiplied by the "down" fingers on my right hand:
1 x 3 = 3
This gets added to the value of the "up" fingers.
So... 9x7 = 63

There is 1 finger down on the left hand, and 3 fingers down on the right hand.

Up fingers: 6 x 10 = 60
Down fingers: 1 x 3 = 3
63

Hence 9 x 7 = 63.

Now try 7 x 6

On your left hand, put 2 fingers up and fold 3 fingers down.

On your right hand, put 1 finger up and fold 4 fingers down.

Up fingers: 3 x 10 = 30
Down fingers: 3 x 4 = 12
42

So 7 x 6 = 42

Our numeration system is based on the number 10 because we have 10 fingers. Many students enjoy using their fingers as a tool for calculation. The idea of having an "ever-ready, built in calculator" gives many students enough confidence to override their memory deficits.

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PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Multiplication of One-Digit and Two-Digit Numbers
Basic Level

MATHEMATICS LEARNING STYLE I

Example: 13
x 4

1000	100	10	1
R	G	B	Y
		O	OO O

← 13

The Chip Trading Model is used to interpret 13. (10 yellows = 1 blue)

1000	100	10	1
R	G	B	Y
		O	OO O
		O	OO O
		O	OO O
		O	OO O

} 4 times

Multiplication is interpreted as repeated addition:

4 x 13 means that a collection of 13 is taken 4 times.

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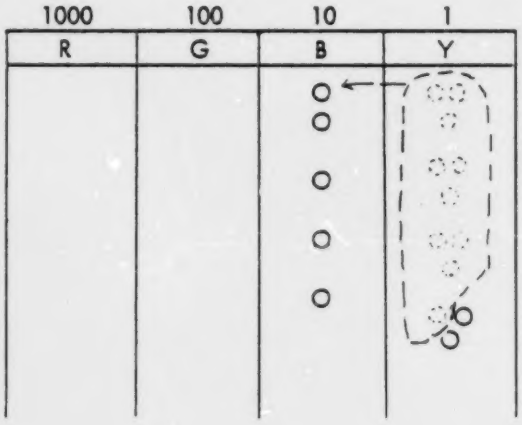
TOPIC: Multiplication of One-Digit and Two-Digit Numbers
Basic Level (Cont'd.)

MATHEMATICS LEARNING STYLE I

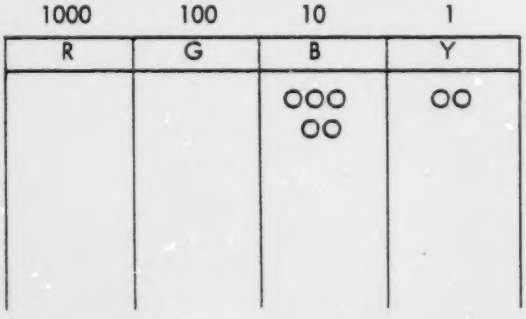
Example: 13
x 4 (Cont'd.)



Trades are made:

10 yellows can be traded for a blue. There are 2 yellows left over.

$$\begin{array}{r} {\scriptstyle 1} \\ 13 \\ \times\ 4 \\ \hline 2 \end{array}$$



In all, there are 5 blues and 2 yellows.

$$\begin{array}{r} {\scriptstyle 1} \\ 13 \\ \times\ 4 \\ \hline 52 \end{array}$$

The answer is 52.

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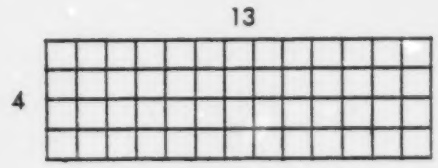
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Multiplication of One-Digit Times Two-Digit Numbers
Basic Level

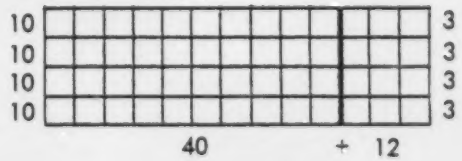
MATHEMATICS LEARNING STYLE II

Example: 13
x 4

Thirteen units (squares or cubes) arranged in a continuous strip.



Multiplication is interpreted as a rectangular array: 4 x 13 means that a rectangle 4 units by 13 units is built. The product is found by counting the total number of units in the area.



Some shortcuts may be used in obtaining the total, such as finding tens and adding on the remaining units. The answer is 52.

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PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Multiplication of One-Digit Times Two-Digit Numbers
Intermediate Level

MATHEMATICS LEARNING STYLE I

Example: 13
x 4

1000	100	10	1
R	G	B	Y
		1	3
		1	3
		1	3
		1	3

The interpretation and manipulation with chips are recorded with numerals. The collection 13 is taken 4 times.

1000	100	10	1
R	G	B	Y
		1	
		1	3
		1	3
		1	3
		1	3
			2

The collections are combined and all possible trades are made. 10 yellows are traded for 1 blue. 2 yellows are left over.

1000	100	10	1
R	G	B	Y
		1	
		1	3
		1	3
		1	3
		1	3
		5	2

13
x 4
13
13
13
13
52

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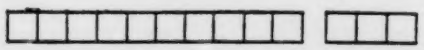
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

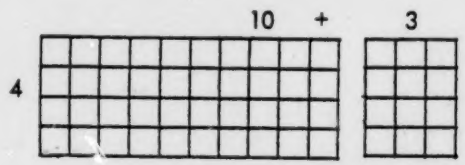
TOPIC: Multiplication of One-Digit Times Two-Digit Numbers
Intermediate Level

MATHEMATICS LEARNING STYLE II

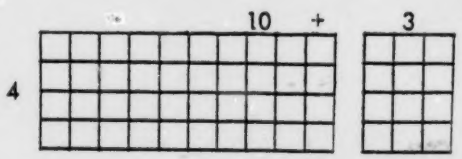
Example: 13
× 4



Thirteen is interpreted by a physical length of 10 (a "long") and 3 units.



4 x 13 is demonstrated as a rectangular region with dimensions 4 x 13.



The distributive property is used to interpret the two separate areas.
4 x (10 + 3)
(4 x 10) + (4 x 3).

13
× 4
→ (4 x 10) + (4 x 3) = 40 + 12 = 52

Numerically,

13
× 4
12 ← 4 x 3
40 ← 4 x 10
52

This method leads naturally to the multiplication algorithm showing the two partial products.

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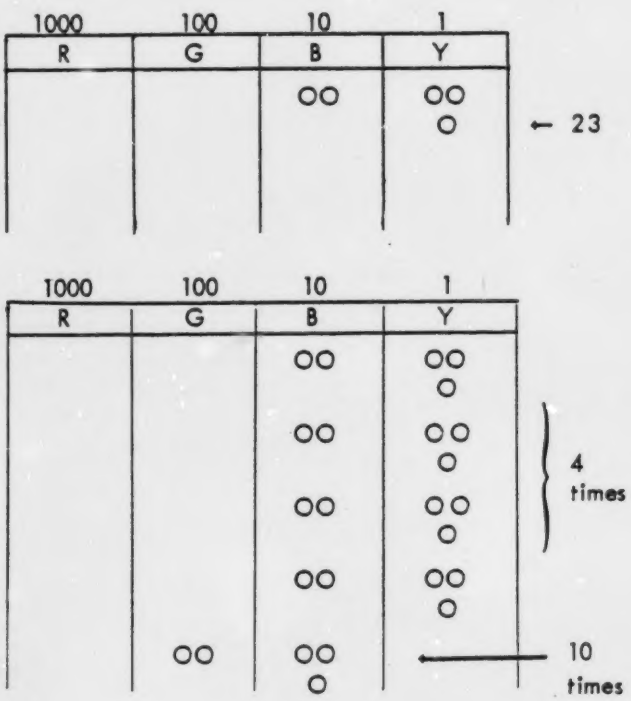
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Multiplication of Two-Digit by Two-Digit Numbers
Basic Level

MATHEMATICS LEARNING STYLE I

Example: 23
× 14



The Chip Trading Model

In Ten-For-One Country, 23 is represented by 2 blues and 3 yellows. The problem of multiplying 23 by 14 means that the collection worth 23 is taken 14 times.

To take the collection 14 times, the student takes the collection "4 times" and then "10 times."

Prior to doing problems like these, students should have discovered the shortcut for taking "10 times" a collection. The shortcut is to shift the collection one place to the left.

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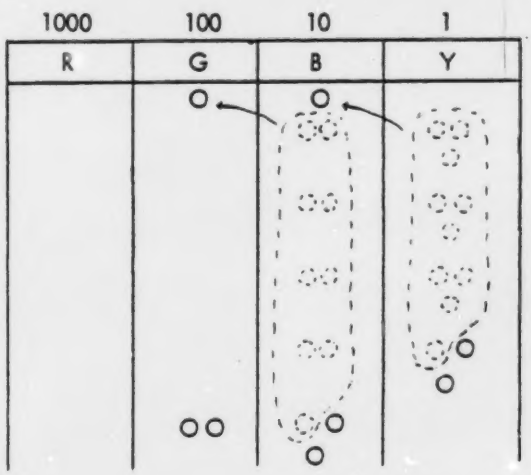
TOPIC: Multiplication of Two-Digit by Two-Digit Numbers
Basic Level (Cont'd.)

MATHEMATICS LEARNING STYLE I

Example: 23
x 14 (Cont'd.)



Now trading is done: 10 yellows can be traded for 1 blue, leaving 2 yellows. Also, 10 blues can be traded for 1 green, leaving 2 blues.

R	G	B	Y
	O OO	O O	OO

The final result is 3 greens, 2 blues, and 2 yellows, or value 322.

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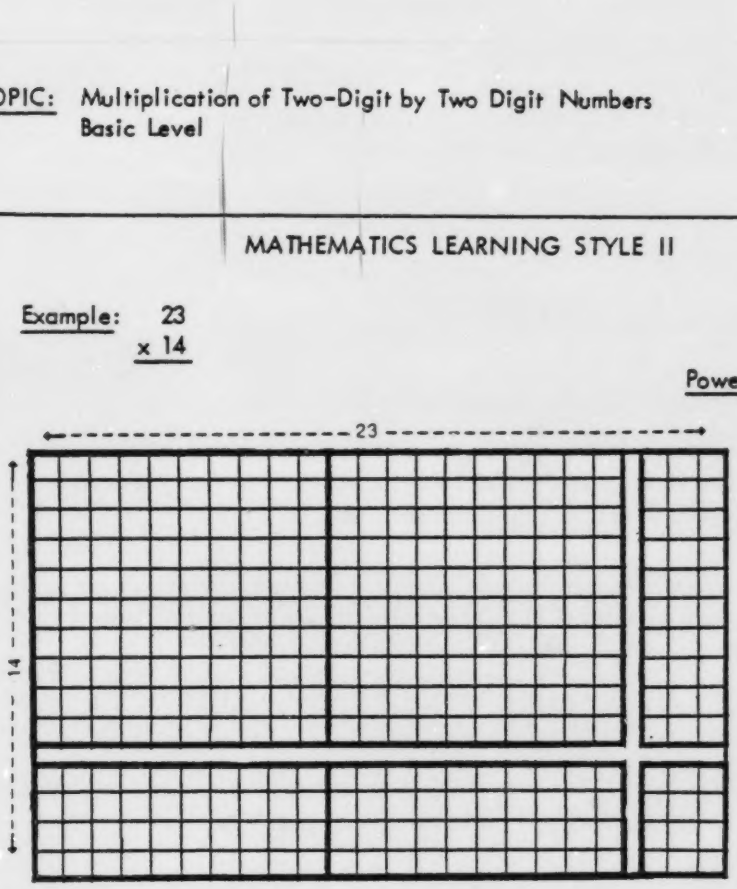
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS TEACHING

TOPIC: Multiplication of Two-Digit by Two Digit Numbers
Basic Level

MATHEMATICS LEARNING STYLE II

Example: 23
x 14

Powers of Ten Model



$14 \times 23 = (10 \times 23) + (4 \times 23)$

Multiplication is interpreted as a rectangular region with its dimensions as the factors to be multiplied, and its area their product.

A rectangular region 14 by 23 is made.

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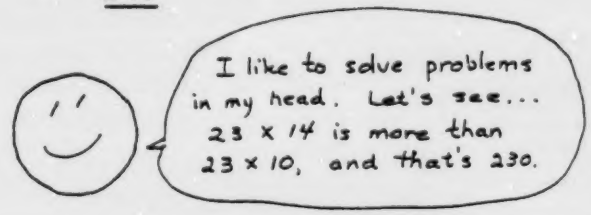
PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR TEACHING MATHEMATICS

TOPIC: Multiplication of Two-Digit by Two-Digit Numbers
Intermediate Level

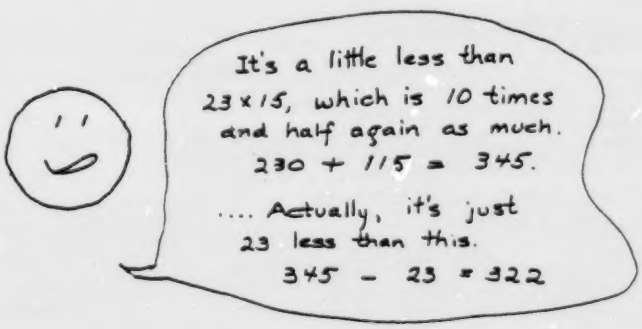
MATHEMATICS LEARNING STYLE II

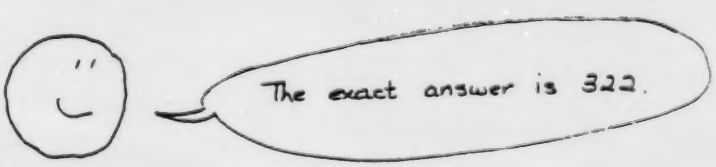
Example: 23
x 14



Mental Arithmetic

An estimation approach.


It's a little less than 23 x 15, which is 10 times and half again as much.
230 + 115 = 345.
.... Actually, it's just 23 less than this.
345 - 23 = 322



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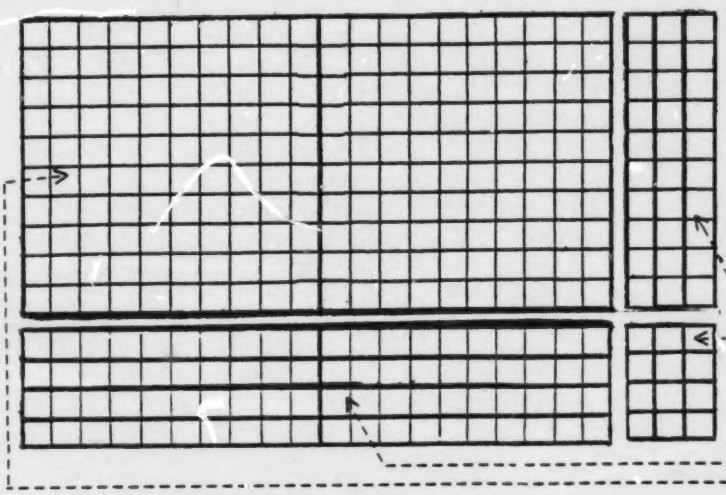
TOPIC: Multiplication of Two-Digit by Two-Digit Numbers
Basic Level (Cont'd.)

MATHEMATICS LEARNING STYLE II

Example: 23
x 14 (Cont'd.)

The region is then covered with the fewest number of pieces of wood. The four separate rectangular regions which result correspond to the four partial products.



23
x 14
12 (4 x 3)
80 (4 x 20)
30 (10 x 3)
200 (10 x 20)

23
x 14
1
12
80
30
200
322

10 of the "longs" can be traded for another "flat"

The partial products are added, and all trades are made.

The final answer is 322.

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PRESCRIPTIVE TEACHING
USING A NEUROBIOLOGICAL FRAMEWORK FOR MATHEMATICS LEARNING

TOPIC: Multiplication of Two-Digit by Two-Digit Numbers
Intermediate Level

MATHEMATICS LEARNING STYLE I

Example:
$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline \\ 23 \\ 23 \\ 23 \\ 23 \\ \hline 230 \end{array}$$
4 times
← 10 times

The interpretation and manipulations with chips are recorded with numerals.

The collection of 2 tens and 3 ones is taken "4 times" and then "10 times."

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline 1 \\ 23 \\ 23 \\ 23 \\ 23 \\ \hline 230 \\ \hline 2 \end{array}$$
10 yellows (ones) are traded for 1 blue (ten) leaving 2 yellows

The shortcut for taking "10 times" the collection has been used. A zero is written to fill the one's place.

The collections are combined, and all possible trades are made.

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline 1 \\ 23 \\ 23 \\ 23 \\ 23 \\ \hline 230 \\ \hline 322 \end{array}$$
10 blues (tens) are traded for 1 green (hundred), leaving 2 blues. There are 3 hundreds in all.

The solution is 322.

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SECTION VI

DISSEMINATION

- RÉSUMÉ OF PATRICIA S. DAVIDSON, PRINCIPAL INVESTIGATOR
- DISSEMINATION: TALKS, WORKSHOPS, AND COURSES ON THE NEUROBIOLOGICAL MODEL AND ITS IMPLICATIONS ON MATHEMATICS EDUCATION, 1979 - 1983

PATRICIA S. DAVIDSON

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Department of Mathematical Sciences
University of Massachusetts
Harbor Campus
Boston, Massachusetts 02125
(617) 929-7960 or 929-7994
(617) 924-6542 (answering service)

EDUCATION

Post Doctoral Study and Research in Neurology and Neuropsychology

Courses and seminars at Harvard Medical School, Columbia University, Institute for the Study of Human Knowledge, New York University Medical School, 1974 - present.

Ed. D. Degree, March 1968, Harvard Graduate School of Education, Program in the Teaching of Mathematics under Professor Edwin E. Moise, 1962-1968.

Dissertation on Learning Problems and Attitudes of Reluctant Learners in Junior High School Mathematics, 1966-1968.

Extensive course work in Mathematics Department, Graduate School of Arts and Sciences, 1962-1966.

Harvard Scholarship Winner, 1962-63, 1964-65.

National Science Foundation Grants, 1962-63, Summer 1963, 1963-64, Summer 1964.

C.A.S., June 1963, Harvard Graduate School of Education, Academic Year Institute for Experienced Science and Mathematics Teachers.

M.A.T. Degree, June 1960, Harvard Graduate School of Education (Radcliffe College), Program in the Teaching of Mathematics; Harvard-Newton Summer Program, Summer 1959.

Pi Lambda Theta Honor Society, 1960
Sloan Foundation Scholarship Winner, 1959-1960

A.B. Degree, June 1959, Middlebury College, Middlebury, Vermont, Mathematics Major, 1955-1959.

Phi Beta Kappa, 1959
Cum Laude, 1959
High Academic Honors in Mathematics,
(Independent Study Project and Honors Thesis) 1959

Edwin Gould Foundation 4-year Scholarship Winner,
(Room, board and tuition) 1955-1959

National and State Elks Scholarship Winner, 1959

Diploma, June 1955, South Royalton High School, South Royalton, Vermont, 1959-1955.

Valedictorian, 1955
Numerous Honors and Awards, 1959-1955

PROFESSIONAL EXPERIENCE

1982 - present — University of Massachusetts, Harbor Campus, Boston, Massachusetts
Professor of Mathematics, Director of Mathematics Resource Center, and Supervisor of Mathematics Tutors.

1970-1982 — Boston State College, Boston, Massachusetts
(Merged with the University of Massachusetts, Boston, January 1982)

Professor of Mathematics, 1974 - January 1982
Associate Professor of Mathematics, 1970-1973

Teaching Duties. Graduate and undergraduate courses in mathematics, methods and content courses for pre-service and in-service elementary teachers, supervision of junior high and high school practice teachers, secondary level teaching seminars, remedial mathematics courses in arithmetic, algebra, and pre-calculus, general education non-mathematics major courses.

Mathematics Resource Center Director. Designed and implemented comprehensive mathematics resource center, planned program, handled

budget, hired and supervised staff assistants, trained staff, and coordinated daily program, 1970-1976.

Curriculum Development. Designed and implemented remedial mathematics programs, 1975 - present. Member of Mathematics Department Curriculum Committee, 1972 - present. Developed Curriculum Guide and Worksheets for District V, Boston Public Schools under Phase II pairing with Boston State College, 1978-1981.

Committee Work. College-wide Development Committee. Committee for Reaccreditation. Faculty Federation Executive Committee. Collective Bargaining Team. Departmental and Divisional Committees on Faculty Evaluation. Affirmative Action Committees for Guideline Preparations and Recruitment. Graduate School Curriculum Committee. College Athletic Budget and Policy Committee.

Grants:

Chapter 636 Grants under Phase II pairing, Boston Public Schools, District V to train teaches and write curriculum; reduced teaching load, 1/4-time, 1978-1981.

National Institute of Education Grant -- Mathematics Learning Viewed from a Neurobiological Model for Intellectual Functioning, (NIE-G-79-0089). Principal Investigator; reduced teaching load, 1/2-time, 1979-1980.

Sabbatical leave for full-time study and research in neurology and neuropsychology related to mathematics learning, under Martha Bridge Denckla, M.D., Children's Hospital Medical Center and Harvard Medical School, 1977-1978.

1974 - present Children's Hospital Medical Center, Boston, Massachusetts

Member of Medical/Educational Team, Learning Disabilities Clinic, Fegan II. Designed and implemented, with Maria Rizzo Marolda, Mathematics Specialist, the Mathematics Diagnostic/Prescriptive Inventory (MDPI), an individualized evaluation instrument to assess math ability, achievement, and learning styles.

Involved in the diagnoses of over 1,200 children, ages 5-21, during a nine-year period. Analyzed clinically and statistically 220 cases

seen over a three-year period under a research grant from the National Institute of Education.

1965-1970 Newton Public Schools, Newton, Massachusetts

City-wide Coordinator of Mathematics, 1967-1970. Coordinated mathematics program for 23 elementary schools and five junior high schools. Worked with high school department heads. Wrote curriculum guidelines. Taught extensively in classrooms, K-9. Conducted in-service workshops for teachers during the school year and summers. Designed and implemented mathematics resource centers and laboratory programs, which served as models for many school systems and teacher centers throughout the country.

1965-1967 Junior High Mathematics Teacher. Taught 7th and 8th grade mathematics at Weeks Junior High. Gathered data for doctoral dissertation. Piloted University of Illinois materials for Slow Learners in Mathematics.

Outstanding Young Woman of America Award,
State Winner for Massachusetts, 1970.

Fulbright Award, Visiting Professorship, Graduate School of Education, Montevideo, Uruguay, January 1969.

Newton Teachers Association Award, 1969.

Co-Founder (with Professor Marion Walter) of Boston Area Mathematics Specialist Groups (BAMS) which now has over 50 members and meets monthly, 1967.

1969-1970 Simmons College, Boston, Massachusetts.

Instructor in Education (part-time). Taught seminar in teaching methods at secondary level and supervised practice teachers.

1964-1969 Harvard Graduate School of Education, Cambridge, Massachusetts.

Associate in Education. Assisted Professor Edwin E. Moise in courses and publications. Supervised intern teachers in 12 school systems. Taught elementary and secondary methods courses. Developed a mathematics curriculum resource center at the Harvard Graduate School of Education.

1962-1963 — Arlington School, McLean Mental Hospital, Belmont, Massachusetts

Mathematics Teacher for Mentally Disturbed Children and Adults (part-time).

1961-1962 — Houghton Mifflin Publishing Company, Mathematics and Science Dept. Assistant Editor, Writer, and Mathematics Consultant (part-time).

1960-1961 — Brookline High School, Brookline, Massachusetts

High school mathematics teacher. Helped develop honors course and piloted School Mathematics Study Groups (SMSG) materials.

1959-1960 — Wayland Junior and Senior High School, Wayland, Massachusetts

Intern teacher under M.A.T. program at Harvard Graduate School of Education.

1957-1959 — Middlebury College, Middlebury, Vermont

Physics Laboratory Instructor

1957-1958 — State Highway Department, Civil Engineering Division, Montpelier, VT

Engineering Aide, Summers 1957, 1958.

SUMMER SCHOOL COURSES — (Details of titles of courses, syllabi, and specific dates are available by request.)

1969 - present — Numerous Graduate Level Summer Courses throughout the country (by invitation), including Bryn Mawr College, University of California at Berkeley, Fairfield University, University of Hawaii, Hayward State College, Lesley College, Johnson (Vermont) State College, Portland (Oregon) State University, Oregon State University, San Diego University, Simmons College.

National Association of Independent Schools Summer Workshops in Mathematics. Taught twelve week-long courses held either in Massachusetts, Pennsylvania, or Texas.

National Institutes on Use of Manipulative Materials. Organized and

taught fifteen week-long courses held in Arizona, California, Connecticut, Georgia, Hawaii, Illinois, Massachusetts, New York, North Carolina, Texas, Vermont, Washington, D.C.

1965-1971 Madison Project Summer Workshops in Berkeley, Chicago, New York, and San Diego.

1969 Graduate Courses in Montevideo, Uruguay, and Bogotá, Colombia.

CONSULTING EXPERIENCE

(Details of places, specific content, and dates are available by request.)

1968 - present Curriculum Consultant to Schools

Conducted workshops, consulted on curriculum writing projects, and led seminars for over 300 public and private schools in many states, including Arizona, California, Colorado, Connecticut, Georgia, Hawaii, Kentucky, Maine, Massachusetts, Michigan, New Hampshire, New Jersey, New York, North Carolina, Pennsylvania, Oregon, Rhode Island, Vermont, Virginia, and Texas; and other countries, including Canada, Colombia, France, Germany, Iceland, New Zealand, Uruguay.

Consultant to Publishers

Reviews and Publishing Ideas for Books: Academic Press, Addison Wesley, Allyn and Bacon, Creative Publications, Cuisenaire Company of America, Houghton Mifflin, National Council of Teachers of Mathematics, Schroedel Vor Schuler, Wadsworth Publishing Company.

Educational Director, Cuisenaire Company of America, Inc., New Rochelle, New York, 1977-1979.

Teacher-Trainer on Hand-Held Calculators. Texas Instruments Educational Division, 1975-1977.

Consultant to National Curriculum Projects

Advisor to Neuroscience Research Projects, Washington State University and Ohio State University.

Review of Project Torque Testing Program, Education Development Center.

Review of National Institute of Education project proposals.

Writing team for National Council of Teachers of Mathematics Yearbook for Elementary Teachers.

Review and Classroom Trials of University of Illinois Project for Slow Learners.

Contributions to Teacher's Guides of Curriculum Projects, Elementary Science Study, Education Development Center.

Staff Member of the Madison Project.

Development of Curriculum for Peace Corps Teacher Training Project.

Development of Curriculum and Teacher Training Workshops for PRIMES Learning Packet Project, State Department of Education, Pennsylvania.

Reviewer of "Infinity Factory" TV Series for Ad Hoc Committee of the National Council of Teachers of Mathematics.

SPEAKING ENGAGEMENTS

1969 - present

Over 500 speaking engagements, an average of three per month, but many more invitations than it is possible to accept.

Ability to speak on a wide variety of topics related to mathematics, mathematics education, neuropsychological issues of learning, and specifically, the neuropsychology of mathematics learning.

Ability to speak to large audiences (500 or more) as well as to communicate with small groups and individuals.

Frequent requests to be a major speaker at national and regional conferences of the National Council of Teachers of Mathematics; conferences of the National Association of Independent Schools; conferences on psychology and the neurosciences related to educational issues; and seminars, workshops, and lectures for public schools, private schools, colleges, universities, and medical schools.

PUBLICATIONS
IN PROGRESS

Spatial Problem Solving with Paper Cutting and Folding, co-authored with Robert E. Willcutt, Cuisenaire Company of America, Inc., 1984.

Mathematics Diagnostic/Prescriptive Inventory, co-authored with Maria R. Marolda. (An individualized diagnostic instrument to assess ability, achievement, and learning styles in mathematics, developed and used over the past nine years to test over 800 children, ages 5 - 21, at the Children's Hospital Medical Center, Boston, Massachusetts.)

The Neuropsychology of Mathematics Learning, co-authored with Maria R. Marolda. (145-page chapter in pre-publication form used in courses and seminars.) To be incorporated into a book to be published by Addison Wesley Publishing Company.

PUBLICATIONS

Spatial Problem Solving with Cuisenaire Rods, co-authored with Robert E. Willcutt, Cuisenaire Company of America, Inc., 1983.

"Mathematics and the Brain: A Tale of Two Hemispheres," by Lorraine Loviglio, (a 5-page article describing my research in The Massachusetts Teacher, January/February 1981).

Mathematics Curriculum Project: Grade 5, Student Worksheets and Teacher's Guide, Leader of Writing Team, Boston Public Schools, District V, 1981.

Mathematics Curriculum Project: Grade 4, Student Worksheets and Teacher's Guide, Leader of Writing Team, Boston Public Schools, District V, 1981.

Mathematics Curriculum Guide, Grades 1-6, Objectives for Coverage and Mastery, Director of Project, Boston Public Schools, District V, 1979.

From Here to There with Cuisenaire Rods, Area, Perimeter, and Volume, co-authored with Robert E. Willcutt, Cuisenaire Company of America, Inc., 1981.

Multiplication and Division with Rod Patterns and Graph Paper, co-authored with Robert E. Willcutt, Cuisenaire Company of America, Inc., 1980.

"Neurobiological Research and Its Implications for Mathematics Education," The Virginia Mathematics Teacher, May 1979.

Picture Puzzles for Cuisenaire Rods, co-authored with Jeffrey B. Sellon, Cuisenaire Company of America, Inc., 1979.

One-Digit Addition and Subrraction using Cuisenaire Rods, Teacher's Resource Book and Student Workbook, Cuisenaire Company of America, Inc., 1978.

Idea Book for the Use of Cuisenaire Rods at the Primary Level, Cuisenaire Company of America, Inc., 1977.

"Rods Can Help Children Learn at All Grade Levels," Learning Magazine, November 1977.

"Mathematics Bibliography for Infinity Factory TV Series," Member of Committee, Bibliography Published for Teachers and Parents, National Council of Teachers of Mathematics, December 1976.

"Evaluation of Infinity Factory TV Series," Member of Evaluation Committee, Report published by the National Council of Teachers of Mathematics, July 1976.

Chip Trading Activities Teacher's Guide, Revised Edition, Scott Resources, Inc., 1975.

Chip Trading Activities, Revised Edition, co-authored with Grace K. Galton and Arlene W. Fair, Scott Resources, Inc., 1975.

Chip Trading Book I, Introduction
Chip Trading Book II, Addition and Subtraction
Chip Trading Book III, Multiplication
Chip Trading Book IV, Division
Chip Trading Book V, Place Value and Numeration
Chip Trading Book VI, Bases Other Than Ten
(Chip Trading Manipulative Materials: Chips, Abacus, Board, Chip Tills, Numeral Cards)

Fraction Bars Program, co-authored with Albert B. Bennett, Scott Resources, Inc., 1973.

Fraction Bars Introductory Card Set
Fraction Bars Card Set I
Fraction Bars Card Set II
Fraction Bars Workbook I
Fraction Bars Workbook II
Fraction Bars Game Mats
(Fraction Bars Manipulative Materials: bars, playing cards, 12-sided dice)

"The Laboratory Approach," co-authored with Marion I. Walter, Chapter 8 of the Thirty-Fifth Yearbook, The Slow Learner in Mathematics, The National Council of Teachers of Mathematics, 1972. Refereed, 61 pages.

Learning Packet Outlines for Mathematics Topics, Grades K-6, PRIMES, Department of Education, Harrisburg, Pennsylvania, 1972.

Chip Trading Activities, co-authored with Grace K. Galton and Arlene W. Fair, Scott Scientific, 1972.

Student Activity Cards for Cuisenaire Rods, co-authored with Arlene W. Fair and Grace K. Galton, Cuisenaire Company of America, Inc., 1971.

Math Bibliography, co-authored with Arlene W. Fair and Grace K. Galton, Growth Activity Products, Inc., 1971. 50 pages.

"A Mathematics Laboratory -- From Dream to Reality," co-authored with Arlene W. Fair, The Arithmetic Teacher, National Council of Teachers of Mathematics, February, 1970. Refereed, 6 pages.

Reprinted as a chapter in Learning Resource Centers, Selected Readings, edited by Neville P. Pearson and Lucius A. Butler, Burgess Publishing Company, 1973.

Reprinted as a chapter in The Mathematics Laboratory, Readings from The Arithmetic Teacher, National Council of Teachers of Mathematics, 1977.

More Topics in Mathematics for Elementary Teachers, Thirtieth Yearbook of the National Council of Teachers of Mathematics, Member of Writing Team.

Also published in a series of separate booklets:
The System of Integers
The System of Rational Numbers
The System of Real Numbers
Logic
Graphs, Relations, and Functions
Informal Geometry
Measurement
Collecting, Organizing, and Interpreting Data
Hints for Problem Solving
Symmetry, Congruence, and Similarity

A series of articles in Professional Growth for Teachers at the Elementary Level, Croft Educational Services, 1969.

"Introducing Mr. Geo"
"Helping Children Understand Number"
"Physical Exchanges with Chips Teach Regrouping"
"Introducing Mathematics Through Pattern"
"Chip Trading - A Strategy for Teaching Place Value"

"An Annotated Bibliography of Manipulative Devices," The Arithmetic Teacher, National Council of Teachers of Mathematics, October 1968. Refereed, 15 pages.

"New Dimensions in Mathematics Instruction at the Elementary and Junior High Levels," Patterns, Newton Public Schools, February 1968.

"Off-Limits," co-authored with Thomas O'Donnell, University of Illinois Newsletter, No. 18, Fall 1966.

Acknowledgments for contributions to: Houghton Mifflin Publishing Company, 1962-1963

Dolciani, Berman, Freilich, Modern Algebra, Book I
Jurgenson, Donnelly, Dolciani, Modern Geometry
Dolciani, Berman, Wooton, Modern Algebra, Book II

ELECTED NATIONAL COMMITTEES

Harvard Graduate School of Education

Alumni Council, 1982-1985
Visiting Committee, 1983-1984

Association of Teachers of Mathematics of New England

Board Member of Eastern Massachusetts Association, 1983-1986
New England Representative, 1983-1986

HOBBIES AND INTERESTS

Extensive travel (throughout the United States, Canada, Europe, and South America), hiking, skiing, gardening, cooking, weaving, tennis, reading, music, theater, and sharing with people.

REFERENCES Furnished upon request.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1983:

Talk for Special Educators, Westchester County Teachers Association, Scarsdale, New York, January 6, 1983.

Consultant for K-12 Mathematics Program, Scarsdale Public Schools, Scarsdale, New York, January 6-7, 1983.

Consultant for K-12 Mathematics Program, Holland Hall School, Tulsa, Oklahoma, January 9, 1983.

Consultant for K-9 Mathematics Program, University School, Shaker Heights, Ohio, January 13-14, 1983.

Consultant for K-12 Mathematics Program, Laurel School, Shaker Heights, Ohio, January 15, 1983.

Lecture to Junior and Senior High Staff in All Fields, Lexington Public Schools, Lexington, Massachusetts, January 18, 1983.

Lecture to Teachers and Administrators in the Education Collaborative of Greater Boston, Concord, Massachusetts, January 20, 1983.

Talk for Parents and Teachers, Milton Academy, Milton, Massachusetts, January 24, 1983.

Attendance at Conference on Neuropsychological Testing Using Luria's Investigations, Teachers College, Columbia University, New York, New York, February 11, 1983.

Workshop for Newton Academic Challenge Program, Newton Public Schools, Newton, Massachusetts, March 3, 1983.

Day-Long Workshop on Problem Solving for National Association of Independent Schools, New York, New York, March 5, 1983.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1983, (cont.)

Workshop for Junior High Mathematics Teachers, Melrose, Massachusetts, March 8, 1983.

Keynote Speaker for Problem Solving Conference, Education Collaborative of Greater Boston, Concord, Massachusetts, March 9, 1983.

Keynote Speaker for Regional Learning Disabilities Conference, Lexington, Massachusetts, April 6, 1983.

Featured Guest Speaker, Learning Institute Mathematics Conference, Long Island, New York, April 9, 1983.

Series of Talks for Gifted Programs, Lawrence, Heath, and Runckle Schools, Brookline, Massachusetts, March - May, 1983.

Consultant for K-12 Mathematics Program, Sidwell Friends School, Washington, D.C., May 3, 1983.

Keynote Speaker and Workshop Leader, Bank Street College Mathematics Conference, New York, New York, May 14, 1983.

Attendance at Four-Day Course on Neuropsychology Related to Schools and Clinics, Teachers College, Columbia University, New York, New York, May 18-21, 1983.

Discussion Leader, University and Secondary School Seminars, University of Massachusetts, Boston, Massachusetts, May, 1983.

Consultant for Holland Hall Schools, Tulsa, Oklahoma, June 1-3, 1983.

Leader of Week-Long Mathematics Workshop and Evening Speaker, National Association of Independent Schools, Dana Hall School, Wellesley, Massachusetts, June 21-24, 1983.

Teacher of Two-Credit Graduate Course on Prescriptive Mathematics, Lesley College, Cambridge, Massachusetts, June 27 - July 28, 1983.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1983, (cont.)

Keynote Speaker and Discussion Leader, Harvard Graduate School of Education Conference on Mid-Career Mathematics Training Program, Cambridge, Massachusetts, July 20-21, 1983.

Teacher of Three-Credit Graduate Course on Mathematics Learning Viewed from a Neuropsychological Perspective, Fairfield University, Fairfield, Connecticut, July 25-29, 1983.

Keynote Speaker and Workshop Leader, Vermont Early Learning Conference, Johnson State College, Johnson, Vermont, August 11, 1983.

Keynote Speaker and Workshop Leader, De Paul Schools for Dyslexia, Louisville, Kentucky, August 25, 1983.

Keynote Speaker and Workshop Leader, Montclair Kimberley Academy, Montclair, New Jersey, September 6, 1983.

Day-Long Workshop, Springer Foundation, Cincinnati, Ohio, October 1, 1983.

Keynote Speaker and Secondary Mathematics Workshop Leader, The Consortium for Staff Development, West Windson- Plainsboro Regional School District, Princeton Junction, New Jersey, October 10, 1983.

Special Interest Speaker for Two Sessions on Giftedness in Mathematics, Regional Conference of the National Council of Teachers of Mathematics, Nashville, Tennessee, October 13-14, 1983.

Speaker on Mathematics Concept Development and the Relationship to Language, Lawrence General Hospital Speech and Hearing Seminar Series, Lawrence, Massachusetts, October 25, 1983.

Workshop on Spatial Problem Solving, Fall Conference of The Association of Teachers in New England, Portland, Maine, October 28, 1983.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1983, (cont.)

Day-Long Workshop, Westport Teachers' Center, Westport, Connecticut, November 5, 1983.

Day-Long Workshop on Logical Reasoning and Thought Processing, National Association of Independent Schools, New York, New York, December 3, 1983.

Member of Massachusetts Task Force, Massachusetts Department of Education, Bureau of Educational Resources, 1983-1984.

Member of National Advisory Board, Educational Technology Center, Harvard Graduate School of Education, Cambridge, Massachusetts, funded under a grant from the National Institute of Education, 1983-88.

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Professional Activities of Patricia S. Davidson during 1982:

Talks for Parents, Weston Public Schools, Weston, Massachusetts, January 7 and January 21, 1982.

Workshop for Teachers, K-6, Gordon School, Providence, Rhode Island, January 8, 1982.

Leader of In-service Course on Neurological Research and Its Implications on Teaching, Scarsdale Public Schools, Scarsdale, New York, January 12-15, 1982.

Workshop, Chestnut Hill School, Newton, Massachusetts, January 25, 1982.

Leader of Staff Development Seminar, Winsor School, Boston, Massachusetts, February 3, 1982.

Consultant and Workshop Leader for Staff Development Seminars, Catlin Gabel School, Portland, Oregon, February 5-6, 1982.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1982, (cont.)

Lecture to Staff and Teachers in Surrounding Communities, Reading Disabilities Research Institute, Rutgers Medical School, Piscataway, New Jersey, February 8, 1982.

Leader of Two-Day Institute, Symposium of the California Association for Neurologically Handicapped Children in Conjunction with the Neurodevelopment Laboratory, Langley Porter Institute, University of California, San Francisco, California, February 19-20, 1982.

Talk to Special Education Staff, Grafton Public Schools, Grafton, Massachusetts, March 1, 1982.

Talk for Parents-Teachers Association, Bishop School, Arlington, Massachusetts, March 11, 1982.

Talk to High School Mathematics Teachers, Grafton Public Schools, Grafton, Massachusetts, March 25, 1982.

Weekend Intensive Course, Fairfield University, Fairfield, Connecticut, March 26-27, 1982.

Speaker, National Meeting of The National Council of Teachers of Mathematics, Toronto, Ontario, Canada, April 15, 1982.

Speaker and Workshop Leader, University School, Shaker Heights, Ohio, April 24, 1982.

Workshop, Newton Academic Challenge Program, Newton Public Schools, Newton, Massachusetts, May 8, 1982.

Speaker at Conference on Issues of Learning, Northeastern University, Boston, Massachusetts, May 27, 1982.

Workshops for K-12 Math Teachers and Special Educators, Scarsdale Public Schools, June 1-2, 1982.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1982, (cont.)

Week-Long Workshop, Independent Schools of the Southwest, Hockaday School, Dallas, Texas, June 14-18, 1982.

Teacher of Three-Credit Graduate Course, Mathematics Department, Portland State University, Portland, Oregon, June 28 - July 2, 1982.

Teacher of Two-Credit Graduate Course, Department of Mathematics Education, Oregon State University, Corvalis, Oregon, July 6-9, 1982.

Talk for Staff at Newton Invervention Early Learning Center, Newton, Massachusetts, July 14, 1982.

Teacher of Two-Credit Graduate Course on Prescriptive Mathematics, Lesley College, Cambridge, Massachusetts, July 18-23, 1982.

Talk and Workshop for Full Faculty, Winsor School, Boston, Massachusetts, September 10, 1982.

Lecture to Editors in All Fields and Workshop for Mathematics Editors, K-12, D.C. Heath Company, Lexington, Massachusetts, September 14, 1982.

Talk for Primary Teachers, Wellesley Public Schools, Wellesley, Massachusetts, September 21, 1982.

Workshop for K-12 Mathematics Teachers, Concord Public Schools, Concord, Massachusetts, September 28, 1982.

Workshop, Teachers of K-2 Gifted and Talented, Newton Public Schools, Newton, Massachusetts, September 30, 1982.

Attendance at Fall Conference of The Association of Teachers of Mathematics in Eastern Massachusetts, Curry College, Milton, Massachusetts, October 1, 1982.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1982, (cont.)

Consultant for K-6 Mathematics Program, St. Bernard's School, New York, New York, October 5, 1982.

Keynote Speaker for 800 Teachers, Conference of The Association of Mathematics Teachers of New York State, Monticello, New York, October 10, 1982.

Guest Lecturer, Visual Thinking Course in Art Department, Taught by Prof. Ron Pollito, University of Massachusetts/Boston, Massachusetts October 12, 1982.

Speaker on Neurological Findings Related to the Learning and Teaching of Science, Greater Boston Biology Club, Needham High School, Needham, Massachusetts, October 21, 1983.

Evening Speaker for Staff and Day-Long Workshop Leader, Doane-Stuart School, Albany, New York, October 22-23, 1982.

Leader for Seminar on Learning Styles for Freshmen Advisors, Harvard University, November 4, 1982.

Speaker, Orton Society Annual Conference, Baltimore, Maryland, November 5, 1982.

General Interest Speak for Two Major Sessions, Regional Conference of National Council of Teachers of Mathematics, Charlotte, North Carolina, November 11-12, 1983.

Day-Long Workshop on Problem Solving, National Association of Independent Schools, New York, New York, November 20, 1982.

Keynote Speaker on Evaluation in Mathematics and General Interest Session Speaker, Regional Conference of the National Council of Teachers of Mathematics, Boston, Massachusetts, December 2-4, 1982.

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Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1981:

Seminar, Psychological Counselors, University of Massachusetts Medical School, Worcester, Massachusetts, February 3, 1981.

Lecture and Workshop, Dalton School, New York, New York, February 6, 1981.

Lecture and Workshop, Collegiate School, New York, New York, February 18, 1981.

Workshop Leader, Annual Conference of Natioanl Association of Independent Schools, Boston, Massachusetts, February 27, 1981.

Speaker and Workshop Leader, Scarsdale Public Schools, Scarsdale, New York, March 6, 1981.

Consultant, Metropolitan Museum Educational Staff, Metropolitan Museum, New York, New York, March 7, 1981.

Speaker, Lincoln Public Schools, Lincoln, Massachusetts, March 9, 1981.

Day-Long Seminar and Workshop, Harvard Graduate School of Education, Cambridge, Massachusetts, September 14, 1981.

Workshop, Andover Public Schools, Andover, Massachusetts, March 25, 1981.

Speaker and Workshop Leader, Spence School, New York, New York, April 3-4, 1981.

Speaker, Brown University Mathematics Department, Providence, Rhode Island, April 13, 1981.

Speaker, Annual Meeting National Council of Teachers of Mathematics, St. Louis, Missouri, April 23, 1981.

Speaker, Neuropsychological Conference, Teachers' College, Columbia University, New York, New York, April 25, 1981.

Dessimination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1981, (cont.)

Speaker, Barnstable School District, Barnstable, Massachusetts, May 23, 1981.

Week-Long Workshop, National Association of Independent Schools, Dana Hall School, Wellesley, Massachusetts, June 22 and 25, 1981.

Leader of Week-Long Workshop, Independent School Association of Southwest, Hockaday School, Dallas, Texas, July 6-10, 1981.

Consultant and Workshop Leader, Mathematics Education Department, Ohio State University, Columbus, Ohio, July 13, 1981.

Teacher of Three-Credit Graduate Course, Johnson State College, Johnson, Vermont, July 20-24, 1981.

Teacher of Two-Credit Graduate Course, Lesley College, July 27-31, 1981.

Speaker at Two Sessions, Revival Conference, Fairfield University, Fairfield, Connecticut, August 3, 1981.

Teacher of Three-Credit Graduate Course, San Diego State College, San Diego, California, August 10-14, 1981.

Day-Long Workshop, Plainfield Public Schools, Plainfield, New Jersey, September 25, 1981.

Speaker at Three Sessions, Conference of Oregon Mathematics Council, Portland, Oregon, October 9-10, 1981.

Speaker, Lincoln Public Schools, Lincoln, Massachusetts, October 15, 1981.

Evening Speaker, Chicago Metropolitan Mathematics Club, Chicago, Illinois, September 16, 1981.

Speaker, Wheelock College, Boston, Massachusetts, October 21, 1981.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1981, (cont.)

Speaker, Lincoln Public Schools, Lincoln, Massachusetts, October 22, 1981.

Speaker and Workshop Leader, Barnstable School District, Barnstable, Massachusetts, October 24, 1981.

Seminar, Lesley College, Cambridge, Massachusetts, October 26, 1981.

Keynote Speaker and Workshop Leader, Greenwich Public Schools Annual Professional Staff Development Day, Greenwich, Connecticut, October 30, 1981.

Speaker, Regional Conference of the National Council of Teachers of Mathematics, Minneapolis, Minnesota, November 13, 1981.

Speaker, Psychology Club, Boston State College, Boston, Massachusetts, November 18, 1981.

Speaker, Lincoln Public Schools, Lincoln, Massachusetts, November 19, 1981.

Workshop for Junior High Staff, Scarsdale Public Schools, Scarsdale, New York, November 20, 1981.

Day-Long Workshop, National Association of Independent Schools, New York, New York, November 21, 1981.

Seminar Leader, Massachusetts Institute of Technology, Cambridge, Massachusetts, November 2, 1981.

Speaker, Lincoln Public Schools, Lincoln, Massachusetts, November 3, 1981.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1980:

Day-Long Workshop, Independent Schools Conference, Poughkeepsie Day School, Poughkeepsie, New York, January 18, 1980.

Keynote Speaker, Mathematics Education Conference, Southwest State University, Marshall, Minnesota, January 26, 1980.

Series of Three Talks, Conference of San Diego Mathematics Council, San Diego, California, February 1-2, 1980.

Speaker, Heads of Lower Schools Conference, New York, New York, February 26, 1980.

Speaker, Regional Conference of the National Council of Teachers of Mathematics, Cherry Hill, New Jersey, March 14, 1980.

Speaker and Workshop Leader, Annual Conference of the National Association of Independent Schools, New Orleans, Louisiana, March 21, 1980.

Speaker and Workshop Leader, Independent Schools of Atlanta, Lovett School, Atlanta, Georgia, March 24, 1980.

Speaker, Regional Conference of the National Council of Teachers of Mathematics, Peoria, Illinois, March 28, 1980.

Speaker, City-wide Staff Meeting, Newton Public Schools, Newton, Massachusetts, April 8, 1980.

Speaker at Two Sessions and Leader of Day-Long Seminar, Annual Conference of the National Council of Teachers of Mathematics, Seattle, Washington, April 17-19, 1980.

Consultant and Workshop Leader, Brearley School, New York, New York, April 30, 1980.

Day-Long Workshop, National Association of Independent Schools, New York, New York, May 10, 1980.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1980, (cont.)

Seminar on Gifted Students, Boston Public Schools, Boston, Massachusetts, May 14, 1980.

Consultant and Workshop Leader, Dalton School, New York, New York, May 21, 1980.

Two-Day Workshop, Conference of North Carolina State Supervisors of Mathematics, June 9-10, 1980.

Week-Long Workshop, National Association of Independent Schools, Dana Hall School, Wellesley, Massachusetts, June 22-27, 1980.

Teacher of Two-Credit Graduate Course, Lesley College, Cambridge, Massachusetts, July 7-11, 1980.

Keynote Speaker, Basic Competency Conference, Vermont State Board of Education, Johnson State College, Johnson, Vermont, August 20, 1980.

Consultant and Workshop Leader, Garrison Forest School, Garrison, Maryland, September 5-6, 1980.

Day-Long Workshop, Plainfield Public Schools, Plainfield, New Jersey, September 26, 1980.

Speaker, Conference of Association of Teachers of Low-Achievers in Mathematics, Salem State College, Salem, Massachusetts, October 16, 1980.

Speaker and Workshop Leader, Conference of North Carolina Mathematics Teachers, Charlotte, North Carolina, October 24, 1980.

Speaker, Conference of Massachusetts Teachers Association, Natick, Massachusetts, October 27, 1980.

Keynote Speaker and Workshop Leader, Greenwich Public Schools Annual Professional Staff Development Day, Greenwich, Connecticut, October 31, 1980.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1980, (cont.)

Day-Long Workshop, National Association of Independent Schools, New York, New York, November 1, 1980.

Speaker, Mathematics Department, San Diego State College, San Diego, California, November 13, 1980.

Speaker and Workshop Leader, Conference of California State Mathematics Council, Anaheim, California, November 14, 1980.

Workshop, Andover Public Schools, Andover, Massachusetts, November 30, 1980.

Speaker, Conference of National Council of Teachers of Mathematics, Washington, D.C., November 22, 1980.

Workshop, Acton Public Schools, Acton, Massachusetts, November 24, 1980.

Speaker and Workshop Leader, Dalton School, New York, New York, December 5, 1980.

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Professional Activities of Patricia S. Davidson during 1979:

Workshop for Middle School Teachers, Buckingham, Browne and Nichols School, Cambridge, Massachusetts, January 12, 1979.

Workshop Leader, Conference of the Association of Teachers of Mathematics in New England, Regis College, Weston, Massachusetts, January 19, 1979.

Day-Long Workshop, Merrimack Public Schools, Merrimack, New Hampshire, January 26, 1979.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1979, (cont.)

Speaker, New York Association of Independent Schools, New York, New York, February 2, 1979.

Sessions Speaker, National Conference of the National Association of Independent Schools, Washington, D.C., March 2, 1979.

Workshop, Acton Public Schools, Acton, Massachusetts, March 8, 1979.

Early Learning Workshop, Cohasset Public Schools, Cohasset, Massachusetts, March 12, 1979.

Speaker and Workshop Leader, Conference of the Association of Teachers in Connecticut, Hartford, Connecticut, March 17, 1979.

Speaker, Regional Conference of the National Council of Teachers of Mathematics, Boston, Massachusetts, April 21, 1979.

Keynote Speaker, Conference of the Association of Support Services, Boston University, Boston, Massachusetts, April 27, 1979.

Day-Long Workshop, National Association of Independent Schools, New York, New York, May 5, 1979.

Day-Long Workshop, Buckingham, Browne and Nichols School, Cambridge, Massachusetts, May 19, 1979.

Workshop Leader, Scarsdale Public Schools, Scarsdale, New York, June 4, 1979.

Workshop, Copley Square High School, Boston Public Schools, Boston, Massachusetts, June 7, 1979.

Leader of two Week-Long Workshops, National Association of Independent Schools, Dana Hall School, Wellesley, Massachusetts, June 18-22, and 25-29, 1979.

Speaker, Wesleyan University, Middletown, Connecticut, July 5, 1979.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1979, (cont.)

Workshop, Andover Public Schools, Andover, Massachusetts, July 11, 1979.

Teacher, Two-Credit Graduate Course, Temple University, Philadelphia, Pennsylvania, July 30 - August 3, 1979.

Day-Long Workshop, Merrimack Public Schools, Merrimack, New Hampshire, August 24, 1979.

Leader of Week-Long Workshop, Lovett School, Atlanta, Georgia, August 27-31, 1979.

Day-Long Workshop, The Doane Stuart School, Albany, New York, September 28, 1979.

Leader of Two-Day Seminar, The Math Center, Washington, D.C., October 19-21, 1979.

Speaker, Conference of the Association of Teachers of Mathematics in New England, Springfield, Massachusetts, November 2, 1979.

Day-Long Workshop, National Association of Independent Schools, New York, New York, November 3, 1979.

Graduate Seminar in Secondary Education, Boston State College, Boston, Massachusetts, November 7, 1979.

Day-Long Workshop, New Canaan Country Day School, New Canaan, Connecticut, November 10, 1979.

Speaker, Mathematics Club, Boston State College, Boston, Massachusetts, November 15, 1979.

Speaker, Maryland Mathematics Conference, College Park, Maryland, November 17, 1979.

Dissemination: Talks, Workshops, and Courses on the Neurobiological Model and Its Implications on Mathematics Education

Professional Activities of Patricia S. Davidson during 1979, (cont.)

Speaker, Newton Public Schools, Newton, Massachusetts, December 4, 1979.

Speaker, Brandeis University, Waltham, Massachusetts, December 8, 1979.

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SECTION VII

PEDAGOGICAL SUGGESTIONS FOR REMEDIATION AND ENRICHMENT

- INTRODUCTION
- COUNTING AND NUMBER WORK
- ARITHMETIC OPERATIONS WITH SPECIAL EMPHASIS ON PLACE VALUE AND CONCEPT DEVELOPMENT
- GAMES AND PUZZLES TO PRACTICE ARITHMETIC CONCEPTS AND OPERATIONS
- GEOMETRIC CONCEPTS
- FRACTION CONCEPTS
- DECIMAL CONCEPTS
- LOGICAL THINKING AND PROBLEM SOLVING
- FURTHER ENRICHMENT ACTIVITIES
- PROBLEM SOLVING AND APPLICATIONS WITH CALCULATORS
- ALGEBRAIC CONCEPTS

SECTION VII

PEDAGOGICAL SUGGESTIONS FOR REMEDIATION AND ENRICHMENT

Recommendations and suggestions for remediation and enrichment are useful only if they reflect the unique nature of the particular learner and are personalized to his or her pattern of strengths and style of learning. In each report emanating from the Mathematics Diagnostic/Prescriptive Inventory, a section on Pedagogical Suggestions is included to aid in the prescriptive aspects of the diagnosis. These Pedagogical Suggestions are presented in the form of books and resource materials which might help a teacher, tutor, or parent build on the strengths and learning style of the individual. Some resource materials are helpful in many cases and are recommended frequently in the personalized prescriptions. This listing is intended to serve as a source of provocation and further investigation. It is not intended to be comprehensive or complete, but only suggestive of potential resources for the special needs of various-aged learners.

This listing of Pedagogical Suggestions is organized into the following content categories:

- Counting and Number Work
- Arithmetic Operations with Special Emphasis on Place Value and Concept Development
- Games and Puzzles to Practice Arithmetic Concepts and Operations
- Geometric Concepts
- Fraction Concepts
- Decimal Concepts
- Logical Thinking and Problem Solving
- Further Enrichment Activities

(Pedagogical Suggestions Contents, Cont.)

- Problem Solving and Applications with Calculators
- Algebraic Concepts

The grade level designations are given in broad categories, taking into consideration both remedial and enrichment needs of very diverse learners. The address of each company is listed just once, on the first designation. For the most part, the major publisher rather than a distributor has been listed for the source, as many items are distributed by numerous companies. In a few instances, only a distributor is mentioned if the publisher is not a usual school supplier. Current catalogs from these various companies will provide additional information on availability of items, quantities and packaging techniques, and shipping policies.

COUNTING AND NUMBER WORK

Children's Arithmetic: The Learning Process by Herbert Ginsburg.
D. Van Nostrand Company, 450 West 33rd Street, New York, N.Y. 10001.

This insightful and very readable book shows how children learn, how their mathematical intuitions develop, and why they might experience difficulty with arithmetic, even at the early stages of counting activities. (Grades K-6)

Idea Book for Cuisenaire Rods at the Primary Level by Patricia S. Davidson.
Cuisenaire Company of America, Inc., 12 Church Street, New Rochelle, N.Y. 10805.

This colorful and pictorial book provides a wealth of beginning ideas for using the continuous (rod) model. The games and activities are carefully sequenced and easy to follow. (Grades K-4)

One-Digit Addition and Subtraction with Cuisenaire Rods by Patricia S. Davidson.
Cuisenaire Company of America, Inc.

This teacher book of games and activities together with a student workbook develop in a sequential fashion the continuous (rod) model for the operations addition and subtraction. (Grades 1-3)

Unifix Introductory Kit and UNIMATH guide by Matthew Scaffa and Janet Castellano.
Creative Publications, P.O. Box 10328, Palo Alto, California 94303.

The Unifix cubes are useful because they can physically represent numbers in either a discrete (sets) or a continuous (rod) model and can be used to show the link between the two models. The guide presents activities dealing with pre-number and number concepts, numeration, operations with whole numbers, estimation, measurement, graphing, fractions, and problem solving. (Grades K-6)

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COUNTING AND NUMBER WORK, (Cont.)

Workjobs: Activity-Centered Learning for Early Childhood Education by Mary Baratta-Lorton. Addison Wesley Publishing Company, Jacobway, Reading, MA 01867.

This popular early childhood book covers in an appealing format 104 single-concept activities in language and mathematics. Presented primarily through a discrete (sets) approach, the math topics include sets, number sequences, combining and separating groups, and relationships.

(Also available is an adapted version, Workjobs For Parents: Activity-Centered Learning in the Home, to help parents extend the learning activities at home.) (Grades K-3)

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ARITHMETIC OPERATIONS WITH SPECIAL EMPHASIS ON PLACE VALUE AND CONCEPT DEVELOPMENT

Base Ten Set
Creative Publications.

or

Powers of Ten
Cuisenaire Company of America, Inc.

This model encourages a continuous (rod) approach to number through etched thousands cubes, etched hundreds squares, etched tens rods, and unit cubes. (The Powers of Ten also come in an ungridded format if preferred.)

The teachers resource book Base Ten Mathematics by Mary Laycock and the student worksheets, Skateboard Practice, Addition and Subtraction, and Skateboard Practice, Multiplication and Division, by Elsie Roberton, Mary Laycock, and Peggy McLean, covering the four operations using base-ten material, would also be helpful and need to be ordered separately from Cuisenaire Company of America, Inc. or from Activity Resources Company, Inc., P.O. Box 4875, Hayward CA 94540. (Grades K-12)

Other similar materials which are not gridded are the Number-Blox by Evelyn Neufeld and James Lucas. Number-Blox Activities: Book A provides activities from sorting and place value to addition and subtraction involving regrouping. Number-Blox Activities: Book B focuses on multiplication and division. Also Picturing Arithmetic -- From Models to Symbols by Robert Madell and Elizabeth Stahl comes in three books, Addition, Subtraction, and Numeration, to be used with Powers of Ten and other base-ten blocks. These are available from Creative Publications. (Grades K-12)

Calendar Math by Helene Silverman and Sheila Siderman.
Cuisenaire Company of America, Inc.

This series of lessons on the calendar involves counting, adding and subtracting, number patterns, and problem solving, and is organized around themes for the twelve months. (Grades 2-6)

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ARITHMETIC OPERATIONS WITH SPECIAL EMPHASIS
ON PLACE VALUE AND CONCEPT DEVELOPMENT, (Cont.)

Chip Trading Activities Program (manipulative materials, student activity books, and teacher's guide) by Patricia S. Davidson, Grace Galton, and Arlene Fair. Scott Resources, Inc., P.O. Box 2121 CZ, Fort Collins, Colorado 80522.

Chip trading helps children learn basic math skills of addition, subtraction, multiplication, division and place value through a wide variety of high interest games and activities. This model encourages a discrete (sets) approach to number. (Grades K-12)

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GAMES AND PUZZLES TO PRACTICE ARITHMETIC CONCEPTS AND OPERATIONS

Deal Me In: The Use of Playing Cards in Teaching and Learning by Margie Golleck. Creative Publications.

> This is a delightful book of more than 80 games and card tricks suitable for learning-disabled children in resource centers or for use in regular classrooms. (Grades 3-12)

Math Card Games by Walter Rucker and Clyde Dilley. Creative Publications.

> There are 41 games organized into six categories to be played with cards: Beginning Concepts, Basic Facts, Computation, Geometry, and Positive/Negative Numbers. (Grades 3-12)

Mathematical Balance and The Balance Book by Lee Jenkins. Activity Resources Company, Inc.,

> By hanging weights on the balance beam, children explore concepts of greater than, less than, addition, subtraction, multiplication, place value, fractions, and logic. (Grades 2-12)

Mathematics Games for Child Involvement by Enoch Dumas. Allyn and Bacon, Inc., 470 Atlantic Avenue, Boston, MA 02210.

> This rich source of games and activities organized by content areas: numeration, addition, subtraction, etc. is extremely adaptable to various ability and age levels. (Grades K-12)

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GAMES AND PUZZLES TO PRACTICE ARITHMETIC CONCEPTS AND OPERATIONS, (Cont.)

Mathematics Their Way by Mary Baratta-Lorton. Addison Wesley Publishing Company.

This exciting book contains 200 classroom-tested experiences and 600 photographs of learning sequences with children using familiar materials such as tiles, buttons, mirrors, bottle caps and jars to teach patterns, comparisons, graphs, measurement, counting, adding, subtracting, problem solving, and place value. (Grades K-6)

Mathimagination, Book A: Beginning Multiplication and Division and Book B: Operations with Whole Numbers by Steve and Janis Marcy. Creative Publications.

These books of worksheets give students the opportunity to work problems which lead to clever built-in rewards and self-checking solutions such as limericks, riddles, and pictures. (Grades 3-12)

Math Mats Starter Kit by Wallace Judd. Creative Publications.

Activities ranging from simple counting games to geometric constructions to be done on highly organized laminated mats give help with the skills of counting, spatial reasoning, pattern perception, numeration, graphing, fractions, ratio, area, and operations. (Grades 2-12)

Multiplication and Division with Rods Patterns and Graph Paper by Patricia S. Davidson and Robert E. Willcutt. Cuisenaire Company of America, Inc.

This 64-page workbook builds understanding of multiplication and division first through one-color rod trains, and then through rod rectangles and "rod tables." The rod patterns are then transferred to more abstract number facts in a variety of puzzles. (Grades 3-8)

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GAMES AND PUZZLES TO PRACTICE ARITHMETIC CONCEPTS AND OPERATIONS, (Cont.)

Picture Puzzles with Cuisenaire Rods by Patricia S. Davidson and Jeffrey Sellon. Cuisenaire Company of America, Inc.

> Self-checking visual worksheets presented in a motivating crossword puzzle format utilize the continuous (rod) model for the understanding of basic arithmetic problems while forming interesting pictures with the rods. (Grades 1-6)

Puzzle Tables -- Number Problems with Computational Skills by Thomas C. O'Brien. Cuisenaire Company of America, Inc.

> This unique set of "puzzle tables" combines problem solving and number drill for addition, subtraction, and multiplication in four levels of difficulty: easy, medium, hard, and challenging. (Grades 3-8)

Student Activity Cards for Cuisenaire Rods by Patricia S. Davidson, Arlene Fair, and Grace Galton. Cuisenaire Company of America, Inc.

> Ten sets of games and activities in the form of motivated activity cards to introduce and practice basic whole number operations and fraction concepts utilize the continuous (rod) model. (Grades 2-12)

Tangle Table by Robert E. Barns. Creative Publications.

> This book of "tangled grids" provides motivated practice with the addition and multiplication tables and is appropriate for remediation at the junior and senior high levels. (Grades 4-12)

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GAMES AND PUZZLES TO PRACTICE ARITHMETIC CONCEPTS AND OPERATIONS, (Cont.)

The Cuisenaire Alphabet Book by Maria Marolda.
Cuisenaire Company of America, Inc.

This clever book uses rods to cover in various ways the letrers of the alphabet and pictures of objects beginning with each letter. The spatial (rod) model is investigated with questions involving counting, addition sentences, early multiplication, and problem solving. (Grades K-6)

Workjobs II: Number Activities for Early Childhood by Mary Baratta-Lorton.
Addison Wesley Publishing Company.

This book is aimed at developing basic skills with numbers and with early arithmetic operations through open-ended activities using the discrete (sets) model. (Grades 3-6)

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GEOMETRIC CONCEPTS

Alphagrams by Teri Perl.
Cuisenaire Company of America, Inc.

Unique tangram-like puzzle activities where students arrange eight geometric pieces to make the 26 letters of the alphabet and to cover pictures of objects beginning with the alphabet letters give experience in spatial reasoning. (Grades K-6)

Designs from Mathematical Patterns by Stanley Bezuszka, S.J., Margaret Kenney, and Linda Silvey. Creative Publications.

Applications of various math topics like modular systems, Pascal's Triangle, and Latin Squares to designs create beautiful patterns which are a blend of arithmetic and geometry. (Grades 6-12)

Dot Paper Geometry -- With or Without a Geoboard by Charles Lund.
Cuisenaire Company of America, Inc.

This book invites an exciting use of the geoboard model for intermediate students. It includes such topics as area, measurement, perimeter, estimation, percent, fractions, and graphing. (Grades 4-8)

Experiences in Visual Thinking by Robert H. Mc Kim.
Creative Publications.

This book on the development of visual perception treats such topics as thinking by visual images, pattern seeking, visual recall, and directed fantasy. (Grades K-12)

Experiencing Geometry by James V. Bruni.
Creative Publications.

This book is an informal and intuitive introduction to geometry developed through observations and experimentation. (Grades 3-12)

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GEOMETRIC CONCEPTS, (Cont.)

Figural Analogies by Howard Black and Sandra Black.
Midwest Publications, P.O. Box 448, Pacific Grove, CA 93950.

Three booklets in progressing levels provide an abundance of non-graded figural analogies to improve visual discrimination and figural reasoning skills. These can be used from gifted elementary to remedial adult education.

Figural Sequences by Howard Black and Sandra Black.
Midwest Publications.

Three booklets in progressing levels provide materials to develop figural reasoning and visual discrimination skills and to promote inductive reasoning. These activities allow primary gifted students and upper elementary-to-adult low reading level students to carry out complex analysis and evaluation tasks.

From Here to There with Cuisenaire Rods - Area, Perimeter, and Volume by Patricia S. Davidson and Robert E. Wilcutt. Cuisenaire Company of America, Inc.

In this 64-page workbook, Cuisenaire rods are used to introduce and reinforce the concepts of area, perimeter, surface area, and volume. Because the rods are metric, teachers have the option of presenting each lesson in metric units. (Grades 3-8)

Geoboard Activity Cards - Primary by Alan Barson.
Scott Resources, Inc.

The primary set contains over 120 nonverbal cards for developmental and perceptual activities and games using the geoboard. Students explore, make shapes and designs, compare similarities and differences, count and work with coordinates, perimeter, and area. (Grades K-4)

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GEOMETRIC CONCEPTS, (Cont.)

Geoboard Activity Cards - Intermediate by Alan Barson.
Scott Resources, Inc.

These 120 cards provide developmental activities, games, and exercises using a geoboard for exploring, counting, area, perimeter, symmetry, fractions, and coordinates. (Grades 3-12)

Geoboard and Geoboard Activity Sheets.
Creative Publications.

These thirty-three spirit duplicating masters provide a carefully sequenced structured set of lessons for a 25-peg geoboard. (Grades 3-12)

Geometry: An Investigative Approach by Phares O'Daffer and Stanley Clemens.
Addison Wesley Publishing Company.

This is a rich and appealing resource book for discovering geometric properties and relating them to real-life situations. It also contains investigations on metric measurement and estimation. (Note that measurement activities provide a valuable means of bridging mathematics to reality and of relating numerical and spatial quantities. There are many metric books and materials available, especially from companies such as Creative Publications and Cuisenaire Company of America, Inc.) (Grades 3-12)

Line Designs by Dale Seymour, Linda Silvey, and Joyce Snider.
Creative Publications.

This easy-to-follow book teaches the techniques of drawing lines to give the illusions of curves in order to create over 100 lovely designs. (Grades 4-12)

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GEOMETRIC CONCEPTS, (Cont.)

Mira and Mira Math for Elementary School.
Creative Publications or Cuisenaire Company of America, Inc.

The plastic Mira, a clever device which reflects images and at the same time can be "seen through," can be used for a host of geometric ideas. The beginning book covers symmetry, congruence, perpendiculars, parallels, angle bisectors, transformations, properties, and drawing techniques. (Grades 4-7)

100 Geometric Games by Pierre Berloquin.
Cuisenaire Company of America, Inc.

Games and puzzles in geometry are clearly stated and answered at the back of the book to provide fun and further understanding of geometric ideas. (Grades 6-12)

Pattern Blocks and Pattern Blocks Activities, Books A and B by Marian Pasternack and Linda Silvey. Creative Publications.

These books include patterns to copy, puzzles to fill with blocks, and tasks to be done, dealing with the concepts of ordering, counting, continuing patterns, symmetry, area, perimeter, congruence, and similarity. (Grades 1-12)

Pattern Blocks Problems for Primary People by Ann Roper and Linda Harvey.
Creative Publications.

This book contains activities with the Pattern Blocks pieces, ranging from the simple copying of designs to filling in patterns where only the number of blocks to be used is given. (Grades K-6)

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GEOMETRIC CONCEPTS, (Cont.)

Perceptual Puzzle Blocks by Dale and Margo Seymour.
Creative Publications.

Carefully sequenced nonverbal activity cards used in conjunction with 9 different blocks provide over 200 puzzles intended to improve visual perception, eye-hand coordination, and problem solving techniques. (Grades 3-12)

Seeing Shapes by Ernest R. Ranucci.
Creative Publications.

This book deals with perspective drawings, symmetry, rotation, reflection, and congruence through many different types of worksheet activities. (Grades 6-12)

Tangramath by Dale Seymour
Creative Publications.

This reproducible workbook involves the use of Tangram puzzles to help students learn in an informal way the concepts of shape, congruence, similarity, perimeter, area, and transformations. (Grades 3-12)

Tangram Patterns by Thomas Foster. Creative Publications.

Nearly 200 tangram puzzles provide patterns for problem solving with tangram pieces. Four different types of puzzles are included. (Grades 1-12)

Tangrams by Ronald C. Read.
Dover Publications, 180 Varrick Street, New York, N.Y. 10014.

This is a rich source book of 330 tangrams problems shown in smaller-than-actual size, and with solutions to make one's own puzzles and activities. (Grades 4-12)

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GEOMETRIC CONCEPTS, (Cont.)

Tangrams, Cards and Materials, developed by Elementary Science Study. Webster/McGraw-Hill, Princeton Road, Hightstown, N.J. 08520.

Three levels of beautifully sequenced activity cards using the set of seven geometric shapes knowr. as "Tangrams" give valuable experience in spat ial problem solving. (Grades K-12)

The Fun With Tangrams Kit by Susan Johnston. Dover Publications.

A compilation of 120 of the best tangram puzzles, including fish, cats, dogs, people, plus a series that tells the story of the house that Jack built. Two sets of tangram pieces and solutions are included. (Grades 3-6)

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FRACTION CONCEPTS

Adventures with Arithmetic Fractions, Book I by Laurence N. Swienciki. Creative Publications.

Stories involving myths and anecdotes from various cultures and times in history are used to generate questions involving fractions through a highly verbal approach. (Grade 6-12)

Everything's Coming Up Fractions with Cuisenaire Rods by John Bradford. Cuisenaire Company of America, Inc.

Through the comparison of two rod lengths, the concept of ratio is used to develop the meaning of fractions, equivalent fractions, addition and subtraction of fractions. Riddles and puzzles are used for motivation and self-checking of answers. (Grades 3-6)

Fraction Bars Program by Albert B. Bennett, Jr. and Patricia S. Davidson. Scott Resources, Inc.

This is a complete program of grames, activities, manipulative materials, worksheets, objectives, and tests for teaching fraction concepts and operations. The Fraction Bars model links both the discrete (sets) approach and continuous (rod) approach. (Grades 3-12)

Fraction Tiles Program by Peggy McLean and Lee Jenkins. Activity Resources Company, Inc.

Bright colored transparent pieces help students visually experience fractional relationships and the four operations. The program includes a manipulative book, student book, and teacher book to pose problems and solutions. (Grades 3-12)

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FRACTION CONCEPTS, (Cont.)

Mathimagination, Book D: Fractions by Steve and Janis Marcy. Creative Publications.

One of six books in a series, this one gives practice on fractions with clever built-in rewards and self-checking solutions such as limericks, riddles, and pictures. (Grades 5-12)

Pattern Blocks pieces and Fractions with Pattern Blocks by Matthew E. Zullie. Creative Publications.

Fraction concepts and the operations are developed with the Pattern Blocks pieces serving as the visual model. (Grades 3-12)

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DECIMAL CONCEPTS

Adventures with Arithmetic Decimals, Book II by Lawrence N. Swienciki. Creative Publications.

Stories involving myths and anecdotes from various cultures and times in history are used to generate questions involving decimals through a highly verbal approach. (Grades 6-12)

Decimal Squares by Albert B. Bennett. Creative Publications.

A new program that teaches decimal skills through the same format of the Fraction Bars. Students use concrete materials and worksheets to progress from the manipulative level to abstract symbols. Games and activities present seven skill areas: readiness, basic concepts, equality, addition, subtraction, multiplication, and division of decimals. (Grades 4-8)

Mathimagination, Book I: Decimals and Per Cents by Steve and Janis Marcy. Creative Publications.

One of six books in a series, this one gives practice on decimals and per cents with clever built-in rewards and self-checking solutions such as limericks, riddles, and pictures. (Grades 5-12)

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LOGICAL THINKING AND PROBLEM SOLVING

Aha! Insight by Martin Gardner, W.J. Freeman, 660 Market St., San Francisco, CA 94104, or Cuisenaire Company of America, Inc.

> This book contains over 60 carefully selected puzzles that encourage creative thinking to improve ability at problem solving. There are six types of problems: combinatorial, geometric, numerical, logical, procedural, and verbal. Each puzzle is answered and discussed in a separate section. (Grades 6-12)

Attribute Games and Activities and Loop Activity Cards by Maria Marolda. Creative Publications.

> This rich teacher's resource guide of games, problems and activities for the Elementary Science Study A-Blocks, Loops, and Label Cards deals with classification and logical relationships between attributes and properties of sets. The physical materials should also be attained. (Grades 1-12)

Basic Thinking Skills by Anita Harnadek. Midwest Publications.

> The purpose of this series of twelve books is to sharpen analytical thinking for better reading comprehension, math, writing, science, test taking, etc. The book titles are: Analogies A, Analogies B, Analogies C, Analogies D, Antonyms and Synonyms, Antonyms, Synonyms, Similarities and Differences, Following Directions - A, Following Directions - B, Miscellaneous, Including Transitivity and Same Person or Not?, Patterns, Think About It, What Would You Do? And True To Life, or Fantasy. The activities are non-graded and can be used from gifted first graders up to adult education.

Developmental Math Activity Cards, A, B, C, D, E, F, G, H, I, J, K, and L by Bates. Addison Wesley Publishing Company.

> Sequenced sets of colorful activity cards promote mathematical thinking through motivated games and activities. (Grades 1-6)

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LOGICAL THINKING AND PROBLEM SOLVING

Finite Differences by Dale Seymour and Margaret Shedd.
Creative Publications.

This important book generates problem solving through functions related to real-life situations and the use of manipulative materials. It gives a clear account of the powerful technique of "finite differences" to solve these problems. (Grades 7-12)

Inductive Thinking Skills by Anita Harnadek.
Midwest Publications.

The purpose of this series of eleven books is to sharpen thinking skills for better reading comprehension, math, writing, science, test taking, etc. The book titles are: Cause and Effect, Figure Patterns, Figure Relations - A, Figure Relations - B, Inferences - A, Inferences - B, Open Ended Problems, Reasoning by Analogy, Relevant Information, Spatial Perception - A, Spatial Perception - B. The activities are non-graded and can be used from gifted first grades up to adult education.

Mathematics, A Human Endeavor by Harold Jacobs.
W.J. Freeman.

This text shows mathematics in its most human form as it unfolds mathematics as having all kinds of patterns. It could serve as a teacher resource guide, as an enrichment text for gifted students, or as a highly motivated book for slower high school and college students. (Grades 6-12)

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LOGICAL THINKING AND PROBLEM SOLVING

Mind Benders (Deductive Thinking Skills) by Anita Harnadek. Midwest Publications.

> The purpose of this series of thirteen books in three levels (easy, medium, hard) is to sharpen deductive thinking skills for better reading comprehension, math, writing, science, test taking, etc. The activities span the range of gifted first graders through adults.

Patterns to Play on a Hundred Chart by Wallace Judd. Creative Publications.

> This book shows more than 44 visual patterns on a hundred chart to help students practice basic computation, problem solving, and pattern recognition. (Grades 3-12)

Successful Problem Solving Techniques by Carole Greenes, John Gregory, and Dale Seymour. Creative Publications.

> This book outlines and describes nine basic techniques for solving problems and demonstrates their use. (Grades 5-12)

Teacher's Guide for Attribute Games and Problems by Elementary Science Study. Webster/McGraw-Hill.

> This is the original teacher resource guide of games, problems, and activities for the Elementary Science Study A-Blocks, which deal with classification and logical relationships between attributes and properties of sets. This guide contains copies of the original Creature Cards. (Grades 1-12)

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LOGICAL THINKING AND PROBLEM SOLVING, (Cont.)

The Psychology of Mathematical Abilities in School Children by V.A. Krutetskii, edited by Jeremy Kilpatrick and Izaak Wirszup. The University of Chicago Press, Chicago, Illinois 60637.

Besides discussing results of much theoretical and experimental research carried out with school children in Russia, this book contains numerous sets and categories of well-planned problems -- arithmetic, algebra, logical, and geometric. (Grades 5-12)

Roddles by Estelle Dickens and Jeffrey Sellon.
Cuisenaire Company of America, Inc.

These sixteen games, puzzles, and Mazes with Cuisenaire rods provide thought-provoking experiences which combine analytical and spatial reasoning. (Grade 4 - Adult)

Visual Logic by Jerzy T. Cwirko-Godycki and Janina E. Karczmarczyk.
Midwest Publications.

This series of four books, Negation, Disjunction, Conjunction, and Three Connectives strengthens thinking skills through colorful logic activities. (Grade 4 - Adult)

Weeple People by Norman Gillespie.
Webster/McGraw-Hill.

This clever little book animates the properties of geometric figures in the style of the Creature Cards from the Attribute Games and Problems. (Grades 4-9)

Wollygoggles and Other Creatures by Thomas C. O'Brien.
Cuisenaire Company of America, Inc.

This book expands the ideas of the Creature Cards developed by Elementary Science Study. Here there are Number Creatures, Number Concept Creatures, Word Creature, and Figural Creatures. Students learn to look for patterns and to establish a systematic approach to solving each problem. (Grades 3-12)

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FURTHER ENRICHMENT ACTIVITIES

Aftermath I - IV by Dale Seymour, et al.
Creative Publications.

This series of fascinating enrichment books with games, puzzles, problems and cartoons is designed to reinforce the regular curriculum and at the same time help students at all levels enjoy discovering mathematics. (Grades 3-12)

Art 'n' Math by Karen Billings, Carol Campbell, and Alice Schwandt.
Cuisenaire Company of America, Inc.

This book translates mathematics into geometric forms and designs, covering spiral patterns, line symmetry, graphing, foldouts, tesselations, and 3-D string designs. (Grades 6-12)

Brain Stretchers, Book 1 and 2 by Carol Anderson and Jackie Haller.
Midwest Publications.

Each of these two books is divided into three categories: logic, patterns, and puzzles. The problems are nongraded and require a great deal of analytical thinking, but little reading. (Grades 3-12).

Classroom Quickies, Book 1, 2, and 3, by Anita Harnadek.
Midwest Publications.

This series of three books is designed to emphasize analytical thinking for those days when your lesson plans do not quite cover the class period. Each problem can be presented within 2 - 10 minutes, but students will not find solutions within this time. (Grades 3-12)

Let's Solve It by Thomas C. O'Brien.
Educational Teaching Aids, 159 West Kenzie Street, Chicago, Illinois 60610.

This is a series of four booklets with motivating problem solving situations appropriate to intermediate grade students. (Grades 4-7)

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FURTHER ENRICHMENT ACTIVITIES, (Cont.)

Math Applications Kit, Science Research Associates, Inc., 155 North Wacker Drive, Chicago, Illinois 60606.

This supplemental activity program gives applications of mathematics in five areas: science, social studies, sports and games, everyday things, and occupations in an attempt to make mathematics relevant and meaningful to students. (Grades 6-9)

Math in Art by Dr. Anne Taylor.
Cuisenaire Company of America, Inc.

This book suggests experiences to discover order in the universe, in nature, in architecture, and in art. Though the children are presented with an art problem, using elements of math, the end products are their own. (Grades 6-12)

The I Hate Mathematics Book by Marilyn Burns.
Creative Publications or Cuisenaire Company of America, Inc.

This delightful book written for students says that mathematics is nothing more than a way of looking at the world. It contains over 200 mathematical events, gags, magic tricks, and experiments to prove it. (Grades 4-12)

The Measurement Book by Sohns and Buffington.
Cuisenaire Company of America, Inc.

This rich metric measurement program is presented in an easy-to-follow format with photographs of children carrying out the activities. (Note that measurement activities provide a valuable means of bridging mathematics to reality, and of relating numerical and spatial quantities. There are many metric books and materials available, especially from companies such as Creative Publications and Cuisenaire Company of America, Inc.) (Grades 4-8)

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PROBLEM SOLVING AND APPLICATIONS WITH CALCULATORS

Calculator Activities for the Classroom, Books 1 and 2 by Georege Immerzeel and Earl Ockenga. Creative Publications.

These two books of student-oriented practice and games for arithmetic and consumer math are designed to help students see the problem-solving capacities of a calculator and become motivated to think their way through solutions to problems. (Grades 4-9)

Calculator Explorations and Problems by Don Miller. Cuisenaire Company of America, Inc.

This book provides a wealth of problems designed for exploring mathematics with the hand calculator, including new ways of looking at computational algorithms, equations, and problem solving. (Grades 4-12)

2,4,6,8, Let's Start to Calculate by Jessica Davidson. Cuisenaire Company of America, Inc.

This book helps students know how to use calculators as well as to learn mathematics. The easy exercises, labeled E, review basic operations with whole numbers and decimals; while the hard exercises, labeled H, provide enrichment in such topics as prime factorization, scientific notation, and signed numbers. (Grades 4-9)

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ALGEBRAIC CONCEPTS

Algebra in the Concrete by Mary Laycock and Reuben Schadler.
Activity Resources Co., Inc.

This pictorial book shows how to build models with manipulative materials for sequences, algebraic expressions, rectangular arrays, and equations. (Grades 7-9)

Graph Gallery by Pat Boyle.
Creative Publications.

This is a richly motivated collection of sheets with sets of ordered pairs, linear equations, and inequalities which, when correctly graphed, compose simple pictures of real-life objects.

Pre-Algebra with Pizzazz by Steve and Janis Marcy.
Creative Publications.

Highly motivated practice leading to clever self-checking rewards is provided on three levels:

Book AA deals with positive and negative integers, fractions, decimals and properties of operations.

Book BB covers the subjects of exponents, decimals, scientific notation, irrational numbers, ratio, proportion, per cent, and probability.

Book CC deals with geometric figures, measurement, square roots, Pythagorean property, trigonometric ratios, and statistics.

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ALGEBRAIC CONCEPTS, (Cont.)

Point-Counterpoint by Ken Rand.
Creative Publications.

This book offers exercises in graphing and reading points: Part I is all first-quadrant graphing, with whole number graphs and additional activities involving estimation, fractions, and decimals. Part II contains similar work in all four quadrants.

Math Tiles, Manual, and Student Labsheets by Peter and Steven Rasmussen.
Key Curriculum Project, P.O. Box 2304, Berkeley, California 94702.

Fascinating manipulative materials are used to show visually such concepts as the multiplying and factoring of polynomials, the distributive property, completing the square, and positive and negative numbers. This program is especially helpful to those students who have greater strengths in spatial and global approaches to learn arithmetic and algebraic processes than in a "recipe" approach.

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SECTION VIII

SELECTED BIBLIOGRAPHY OF RESEARCH ON THE BRAIN AND INTELLECTUAL FUNCTIONING

- INTRODUCTION
- SELECTED BIBLIOGRAPHY OF RESEARCH ON THE BRAIN AND INTELLECTUAL FUNCTIONING

SECTION VIII

SELECTED BIBLIOGRAPHY OF RESEARCH ON THE BRAIN AND INTELLECTUAL FUNCTIONING

For those interested in pursuing further the neuropsychological model for learning, this bibliography is presented as a sample of the types of readings available at the present time. It should be noted that most of the studies on the intellectual functioning of the brain have been in the general area of learning or in the areas of reading and language (with very little written specifically about mathematical learning), and that most of the books and articles published thus far stem from research with adult neurological patients rather than with children. Furthermore, in the history of research in medicine, studies have often centered on unique populations who have obvious deficits, whereby the style of research associates the observed deficits in terms of known physical conditions. By implication, the physical conditions that seem to affect the loss of effective function suggest that effective function is associated with the corresponding "healthy" physical aspect.

By its very nature and because of the increased research activity in all areas of brain research, this list of readings should serve only as a cursory, ephemeral review, subject to constant refinement and revision. It also should be noted that many of the sources listed here contain further bibliographical references for each specialized topic.

Also included in this bibliography are some general reference books on mathematics learning which focus on issues of diagnoses and teaching strategies for various populations of students, including learning disabled and special needs children.

Selected Bibliography of Research on the Brain and Intellectual Functioning

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(A four-page bulletin published every three weeks, giving highlights on current research, theory, and practice on various aspects of neuropsychology, neurology, and psychology.)

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